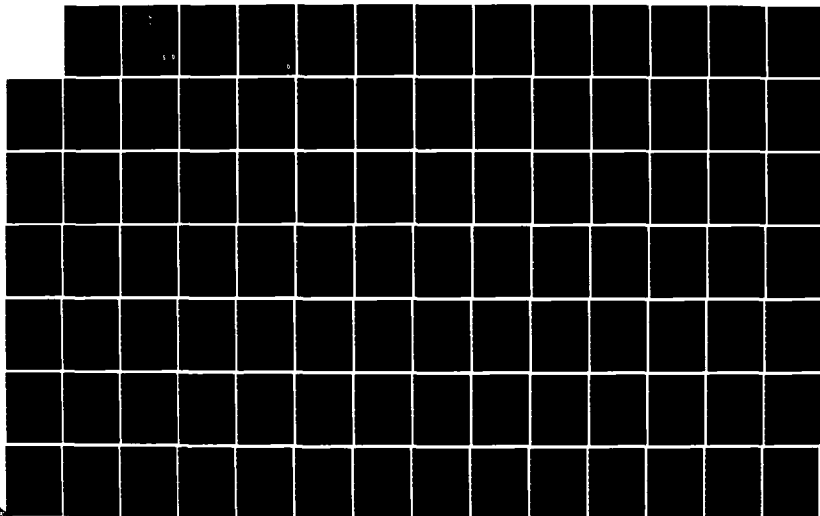


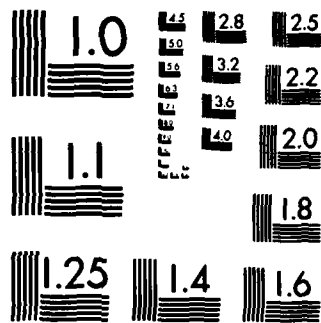
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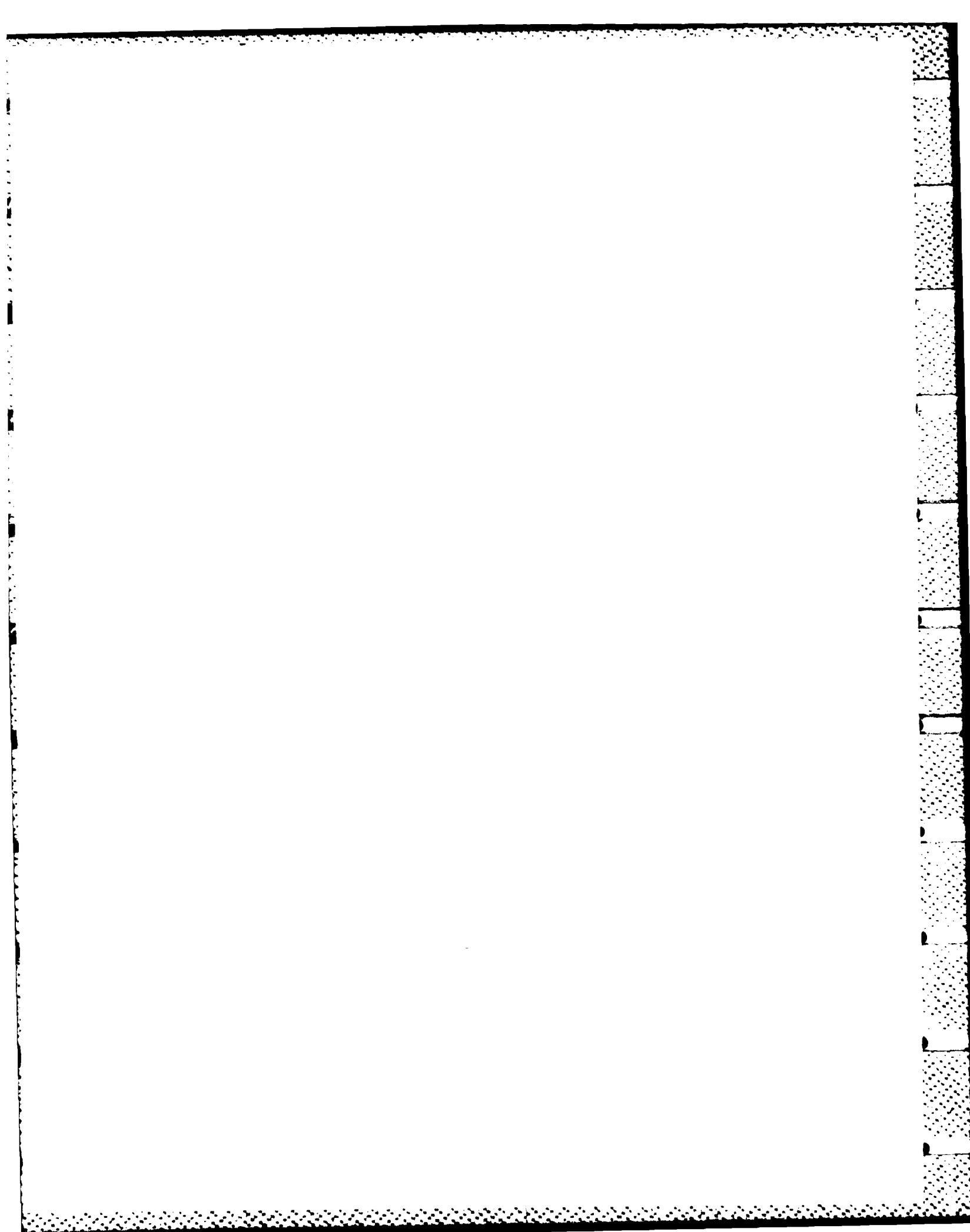
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# FLIGHT CONTROL SYSTEM RECONFIGURATION DESIGN USING QUANTITATIVE FEEDBACK THEORY

# THESIS

Presented to the Faculty of the School of Engineering  
of the Air Force Institute of Technology  
Air University  
in Partial Fulfillment of the  
Requirements for the Degree of  
Master of Science

Phillip B. Arnold, B.S.E.E.  
Major USAF  
Graduate Electrical Engineering  
December 1984

Approved for public release; distribution limited

## Preface

I extend my sincere thanks to my thesis advisors Professors Constantine H. Houpis, Isaac Horowitz, and John J. D'Azzo for their cooperation and patience throughout this thesis effort. Their help contributed immensely to the successful completion of this thesis. A special acknowledgement is given to Mr. Yin-Kuei Liao of the Chung-Shan Institute of Science, Republic of China. His expert assistance given so freely and tirelessly is greatly appreciated.

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significantly impacts LCC as actuators are 50 percent of FCS LCC. Correspondingly, simpler actuators reduce maintenance actions required (4).

Quantitative feedback theory is ideally suited for the FCS reconfiguration problem. In QFT the design is accomplished a priori so that the system output is in the acceptable set of outputs for all possible members of the plant set. Surface failures simply increase the number of equations in the equivalent plant set, as is seen in Chapter III. The transparency of QFT enables the designer to readily see the extra "cost of feedback" for this enlargement of the plant equation set by including surface failures (5). A QFT design approach also reduces the need for complex fault identification and isolation (FDI) schemes.

## I.2 Problem

The main purpose of this thesis effort is to investigate the usefulness of QFT in designing for FCS reconfiguration. The minimum phase plant equations are based upon linearized equations of motion for the AFTI/F-16. Non-minimum phase plant equations are not covered in this thesis. Using QFT, control laws are developed to achieve specified performance tolerances for several flight control surface failures over a large portion of the flight envelope. Pitch rate ( $q$ ) and roll rate ( $p$ ) are the two output variables to be controlled. Control surfaces are

In this thesis, QFT is applied to a FCS reconfiguration problem in an attempt to gain insight into its utility as a design tool.

The remainder of the first chapter presents pertinent background information, a statement and scope of the problem, the assumptions used, the approach taken, and the sequence of presentation for the thesis.

## I.1 Background

Several promising benefits of inherent reconfiguration have been detailed in Self-Repairing Flight Control System Reliability and Maintainability, Program Plan (4). The primary benefits to be derived from inherent reconfiguration include improvements in reliability and maintainability (R&M), survivability, and reduced life cycle costs (LCC).

Improvements in R&M are expected as a result of fewer and less complex FCS components. Survivability will be improved by taking advantage of existing redundant control surfaces, thereby reducing the dependency on any one set of control surfaces for primary control about an axis. The reduction in criticality of the control surfaces allows a reduction in the redundancy level at the actuator. The actuator no longer becomes the redundancy integrator and the actuator then can be made simple and less complex. Simplex or dual actuators can be contemplated which



FLIGHT CONTROL SYSTEM RECONFIGURATION DESIGN  
USING QUANTITATIVE FEEDBACK THEORY

I. Introduction

Design planning for the next generation fighter aircraft is presently in progress. The aircraft, known as the Advanced Tactical Fighter (ATF), will have a flight control system (FCS) capable of taking advantage of the inherent redundancy existing in redundant flight control surfaces. This means that no single loss or failure of a FCS component will result in loss of aircraft control. A digital flight control system will redistribute control authority to the remaining components to generate the necessary aerodynamic forces and moments. This reconfiguration of control authority by an aircraft with inherent redundancy gives rise to the term "inherent reconfiguration."

Methods of developing reconfiguration schemes are now being investigated by the Flight Dynamics Laboratory (1). One promising approach is quantitative feedback theory (QFT) as developed and successfully applied to a number of difficult problems by Dr. Isaac Horowitz (2; 3). Quantitative feedback theory is ideally suited to develop robust control laws for systems with highly uncertain parameters.

Fixed-compensators and prefilters are synthesized for the pitch rate and roll rate channels, and a digital simulation is conducted at four points in the flight envelope with up to two simultaneous surface failures, as well as, for the unimpaired aircraft. Results of the simulation show that the compensators provide robust control for each control surface configuration at each flight condition. Compensator designs are also synthesized to provide only stability when three of the four surfaces are failed.

The theory developed by Dr. Horowitz is found to be effective in designing for robustness despite large uncertainty arising from control surface failures, and plant parameter variation. Flight control research using quantitative feedback theory should be continued. The fixed-compensator design approach has potential to significantly reduce the complexity of a reconfigurable flight control system. A computer-aided design package for quantitative feedback theory should be developed to facilitate design work.

Abstract

Quantitative theory developed by Dr. Isaac Horowitz of the Weizmann Institute of Science, Israel is used to develop control laws for the AFTI/F-16 with a reconfigurable flight control system. Compensators are synthesized to control pitch rate and roll rate through individually controlled elevators and flaperons. Robust control of these variables is required over a large portion of the flight envelope despite flight control surface failures.

Linearized aerodynamic data are used to develop the aircraft model in state-variable format. The longitudinal and lateral-directional equations are coupled in the control matrix. Individual control of the elevators and flaperons is obtained by dividing the dimensionalized control derivatives for a control surface pair in half and assigning each surface of the pair one-half of the total derivative value. The system with individually controlled surfaces represents a four input-two output system which is transformed into an equivalent two input-two output system for each control surface configuration and flight condition. Quantitative feedback theory is then applied to the equivalent systems.

$\theta$       -- Pitch angle  
 $\phi$       -- Roll angle  
 $\mu_{ij}$     -- Control surface constants  
 $\omega$       -- Frequency

$w$       -- Weight  
 $\underline{x}$       -- State vector  
 $X_\theta$      -- Dimensional variation of x-force with pitch angle  
 $X_u$      -- Dimensional variation of x-force with forward velocity  
 $X_\alpha$     -- Dimensional variation of x-force with angle-of-attack  
 $X_q$      -- Dimensional variation of x-force with pitch rate  
 $X_\delta$     -- Dimensional variation of x-force with elevator ( $\delta_1$ ) and flaperon ( $\delta_2$ )  
 $Y_\phi$     -- Dimensional variation of y-force with roll angle  
 $Y_\beta$     -- Dimensional variation of y-force with sideslip angle  
 $Y_p$     -- Dimensional variation of y-force with roll rate  
 $Y_r$     -- Dimensional variation of y-force with yaw rate  
 $Y_\delta$     -- Dimensional variation of y-force with elevator ( $\delta_1$ ) or flaperon ( $\delta_2$ )  
 $Z_\theta$     -- Dimensional variation of z-force with pitch angle  
 $Z_u$     -- Dimensional variation of z-force with forward velocity  
 $Z_\alpha$     -- Dimensional variation of z-force with angle-of-attack  
 $Z_q$     -- Dimensional variation of z-force with pitch rate  
 $Z_\delta$     -- Dimensional variation of z-force with elevator ( $\delta_1$ ) or flaperon ( $\delta_2$ )  
 $\delta_1$     -- Elevator input  
 $\delta_2$     -- Flaperon input  
 $\delta_r$     -- Rudder input  
 $\delta_c$     -- Canard input

$M_p$  -- Peak value  
 $M_\theta$  -- Dimensional variation of pitching moment with pitch angle  
 $M_u$  -- Dimensional variation of pitching moment with forward velocity  
 $M_\alpha$  -- Dimensional variation of pitching moment with angle-of-attack  
 $M_q$  -- Dimensional variation of pitching moment with pitch rate  
 $M_\delta$  -- Dimensional variation of pitching moment with elevator ( $\delta_1$ ) or flaperon ( $\delta_2$ )  
 $N_\beta$  -- Dimensional variation of yawing moment with side-slip angle  
 $N_p$  -- Dimensional variation of yawing moment with roll rate  
 $N_r$  -- Dimensional variation of yawing moment with yaw rate  
 $N_\delta$  -- Dimensional variation of yawing moment with elevator ( $\delta_1$ ) or flaperon ( $\delta_2$ )  
 $p$  -- Roll rate  
 $\underline{P}_e$  -- Equivalent plant matrix  
 $P_{ij}$  -- Basic plant equations  
 $q$  -- Pitch rate  
 $Q_{ij}$  -- Plant equations for equivalent single input-single output systems  
 $r$  -- Yaw rate  
 $s$  -- Wing reference area  
 $\underline{T}$  -- System transfer function matrix ( $t_{ij}$ )  
 $t_s$  -- Settling time  
 $t_r$  -- Rise time  
 $u$  -- Forward velocity

### List of Symbols

$\alpha$	-- Angle-of-attack
$\underline{A}$	-- Plant matrix
$\underline{B}$	-- Control matrix
$b_{ij}$	-- Performance specification bounds
$b$	-- Wing span
$\beta$	-- Sideslip angle
$\bar{C}$	-- Wing aerodynamic chord
Ft	-- Feet
lbs	-- Pounds
$g_1$	-- Pitch rate compensator transfer function
$g_2$	-- Roll rate compensator transfer function
$f_{ij}$	-- Prefilter
$I_{xx}$	-- Moment of inertia about x-axis
$I_{yy}$	-- Moment of inertia about y-axis
$I_{zz}$	-- Moment of inertia about z-axis
$I_{xz}$	-- Product of inertia in xyz system
$L_\beta$	-- Dimensional variation of rolling moment with sideslip angle
$L_p$	-- Dimensional variation of rolling moment with roll rate
$L_r$	-- Dimensional variation of rolling moment with yaw rate
$L_\delta$	-- Dimensional variation of rolling moment with elevator ( $\delta_1$ ) or flaperon ( $\delta_2$ )
$L_{10}$	-- Pitch rate nominal loop transmission
$L_{20}$	-- Roll rate nominal loop transmission

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two flaperons and two horizontal tails each one individually controlled. Control surface configurations are as follows:

Mode 1 - All four surfaces are operating normally.

Mode 2 - One horizontal tail is failed.

Mode 3 - One flaperon is failed.

Mode 4 - One horizontal tail and one flaperon are failed (same side).

Mode 5 - One horizontal tail and one flaperon are failed (opposite sides).

Mode 6 - Both flaperons are failed.

Throughout this thesis, modes 1 through 6 are referred to as control surface configuration (CSC) modes, and a case refers to a particular flight condition and CSC mode. Also, for the purposes of this study, a failed surface is defined to be fixed in the neutral reference position.

A large degree of parameter uncertainty is incorporated into the design by choosing four design points representing a large portion of the flight envelope. These design points are seen in Figure 1-1.

### I.3 Assumptions

The aircraft equations of motion were developed by Mr. Finley Barfield using the assumptions listed below (6).

1. The aircraft is a rigid body, and mass is constant.

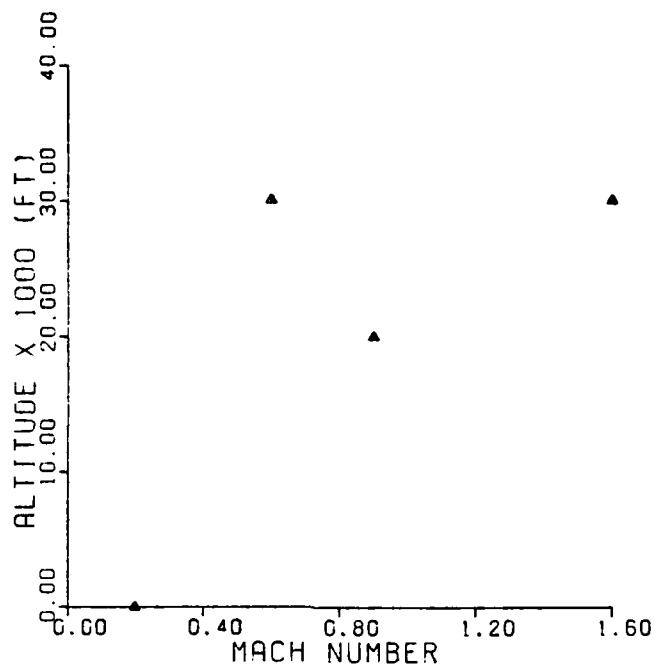


Fig. 1-1. Flight Envelope Design Points

2. Thrust is not changing.
3. The earth's surface is an inertial reference frame.
4. The atmosphere is assumed fixed with respect to the earth.
5. The equations of motion can be decoupled into a longitudinal and a lateral-directional set.
6. Linearization about an operating condition is acceptable for point designs.
7. Aerodynamics are fixed for Mach number and altitude.

A detailed explanation of each of the above assumptions is presented in Reference 6.

In addition to the above assumptions, the lateral and longitudinal control derivatives for the elevators and flaperons are halved in order to obtain control coefficients for the individual control surfaces. This approach is assumed suitably accurate for a preliminary design study of this nature.

#### I.4 Approach

The first step in the design process is to derive the basic plant transfer functions. These are obtained from the aircraft equations in state-variable form, with the longitudinal and lateral control elements coupled in the input matrix. From the system block diagram, equivalent plant equations are derived for each of the previously described surface configurations. As described in Chapter III, the four input-two output system is transformed into an equivalent two-by-two system. The resulting two-by-two equivalent plant transfer function matrices,  $\underline{P}_e$ , are then inverted to yield reciprocal expressions,  $\underline{P}_e^{-1} = (1/Q_{ij})$ , for equivalent single input-single output (SISO) systems. Next, output specifications are formulated.

The pitch rate and roll rate specifications are first developed in the time domain, and then transformed into the frequency domain via the Laplace transformation. Designing in the frequency domain avoids the more difficult convolution operations of time functions. The design specifications serve mainly to demonstrate the application of

the technique. They are based upon what the author feels are reasonable from his personal experience gained from flying fighter aircraft, and serve primarily as a basis for demonstrating QFT. Next, the compensators are synthesized according to the techniques of Reference 3.

The longitudinal and lateral controller designs are incorporated into the complete system, and the design is verified by simulations at each flight condition for each control surface configuration. Control surface deflections are also examined for position and rate saturation.

#### I.5 Presentation

This thesis consists of six chapters. Chapter II contains the aircraft equations of motion in state variable format that are used to derive the basic plant transfer function equations. Chapter III presents the system model used in the simulation and the quantitative feedback synthesis theory applied to the reconfiguration problem. The actual design of the control laws is presented in Chapter IV. Chapter V displays the output and control surface responses, and evaluations of such. Chapter VI contains the conclusions and recommendations.



## II. Aircraft

### II.1 Introduction

In this chapter the decoupled longitudinal and lateral aircraft equations of motion in state-variable format are coupled together in the input matrix allowing control of individual elevators and flaperons. From these coupled aircraft equations, the basic plant transfer functions of interest are computed. The linearized, aerodynamic coefficients used to develop the equations of motion are developed by the General Dynamics Corporation, and are taken from Reference 6. A description of the AFTI/F-16 and its flight control system is also given in Reference 6. The aerodynamic coefficients used in the equations are listed in Appendix A for four flight conditions. Table 2-1 presents pertinent aircraft data for the AFTI/F-16.

### II.2 Aerodynamic Model

The equations of motion are written in state-space form,  $\dot{\underline{x}} = \underline{A}\underline{x} + \underline{B}u$ . They are a set of first-order differential equations which represent the aircraft dynamics. The elements of the A and B matrices are dimensionalized body axis coefficients. Coefficients in the stability axis system are converted to the body axis system using a computer program called Conversion and Transformation (6).

TABLE 2-1  
AFTI/F-16 AIRCRAFT DATA

---

---

Aircraft Parameters

Wing Mean Aerodynamic Chord,  $c(\text{ft}) = 11.32$

Wing Reference Area,  $s(\text{ft}^2) = 300.0$

Wing Span,  $b(\text{ft}) = 30.0$

Weight,  $w(\text{lbs}) = 21,018.0$

Moment of Inertias

$I_{xx}(\text{slug-ft}^2) = 10,033.4$

$I_{yy}(\text{slug-ft}^2) = 53,876.3$

$I_{zz}(\text{slug-ft}^2) = 61,278.4$

$I_{xz}(\text{slug-ft}^2) = 282.132$

---

The body axis is chosen as the design axis for two primary reasons. First, the sensors providing feedback signals actually measure body accelerations and rates. Secondly, the pilot desires to control accelerations and rates that he actually feels which are body accelerations and rates (6).

Using the assumptions of Chapter I, and assuming that coupling can be ignored between the longitudinal and lateral axes, two sets of linear, first-order differential equations result. The longitudinal set of equations has the following form:

$$\begin{bmatrix} \dot{\theta} \\ \dot{u} \\ \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ X_{\theta}' & X_u' & X_{\alpha}' & X_q' \\ Z_{\theta}' & Z_u' & Z_{\alpha}' & Z_q' \\ M_{\theta}' & M_u' & M_{\alpha}' & M_q' \end{bmatrix} \begin{bmatrix} \theta \\ u \\ \alpha \\ q \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ X_{\delta e}' & X_{\delta f}' \\ Z_{\delta e}' & Z_{\delta f}' \\ M_{\delta e}' & M_{\delta f}' \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_f \end{bmatrix} \quad (2-1)$$

where

$\theta$  = pitch angle (deg)

$u$  = forward velocity (ft/sec)

$\alpha$  = angle-of-attack (deg/sec)

$q$  = pitch rate (deg/sec)

The lateral set of equations has the following form:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\beta} \\ \dot{p} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ Y_{\phi}' & Y_{\beta}' & Y_p' & Y_r' \\ 0 & L_{\beta}' & L_p' & L_r' \\ 0 & N_{\beta}' & N_p' & N_r' \end{bmatrix} \begin{bmatrix} \phi \\ \beta \\ p \\ r \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ Y_{\delta a}' & Y_{\delta r}' & Y_{\delta c}' \\ L_{\delta a}' & L_{\delta r}' & L_{\delta c}' \\ N_{\delta a}' & N_{\delta r}' & N_{\delta c}' \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \\ \delta_c \end{bmatrix} \quad (2-2)$$

where

$\phi$  = bank angle (deg)

$\beta$  = sideslip angle (deg)

$p$  = roll rate (deg/sec)

$r$  = yaw rate (deg/sec)

Using the computer program Total, transfer function expressions for the desired output-to-input relationships are computed (7). These transfer functions are referred to as the basic plant equations. They are identified by a two-digit subscript, and a superscript (a or b). The first digit in the subscript denotes the output while the second digit denotes the control surface input that generates the output. Pitch and roll rates are identified by a 1 or a 2, respectively. Likewise, the elevators and flaperons are identified by a 1 or a 2, respectively. A superscript "a" or "b" denotes a right or left control surface, respectively. For example,  $P_{12}^a$  identifies the basic plant equation whose output is pitch rate generated by the right flaperon. Combining the longitudinal and lateral matrices, and dividing control between individual elevator and flaperon surfaces results in a set of equations with coupling between the longitudinal and lateral axes in the control matrix (Eq. 2-3).

$$\begin{bmatrix} \dot{\theta} \\ \dot{u} \\ \dot{\alpha} \\ \dot{q} \\ \dot{\phi} \\ \dot{\beta} \\ \dot{p} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ X_{\theta}' & X_u' & X_{\alpha}' & X_q' & 0 & 0 & 0 & 0 \\ Z_{\theta}' & Z_u' & Z_{\alpha}' & Z_q' & 0 & 0 & 0 & 0 \\ M_{\theta}' & M_u' & M_{\alpha}' & M_q' & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & Y_{\phi}' & Y_{\beta}' & Y_p' & Y_r' \\ 0 & 0 & 0 & 0 & 0 & L_{\beta}' & L_p' & L_r' \\ 0 & 0 & 0 & 0 & 0 & N_{\beta}' & N_p' & N_r' \end{bmatrix} \begin{bmatrix} \theta \\ u \\ \alpha \\ q \\ \phi \\ \beta \\ p \\ r \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2}X_{\delta_e}' & \frac{1}{2}X_{\delta_e}' & \frac{1}{2}Y_{\delta_f}' & \frac{1}{2}Y_{\delta_f}' & 0 & 0 \\ \frac{1}{2}Z_{\delta_e}' & \frac{1}{2}Z_{\delta_e}' & \frac{1}{2}Z_{\delta_f}' & \frac{1}{2}Z_{\delta_f}' & 0 & 0 \\ \frac{1}{2}M_{\delta_e}' & \frac{1}{2}M_{\delta_e}' & \frac{1}{2}M_{\delta_f}' & \frac{1}{2}M_{\delta_f}' & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2}Y_{\delta_e}' & -\frac{1}{2}Y_{\delta_e}' & \frac{1}{2}Y_{\delta_a}' & -\frac{1}{2}Y_{\delta_a}' & Y_{\delta_r}' & Y_{\delta_c}' \\ \frac{1}{2}L_{\delta_e}' & -\frac{1}{2}L_{\delta_e}' & \frac{1}{2}L_{\delta_a}' & -\frac{1}{2}L_{\delta_a}' & L_{\delta_r}' & Y_{\delta_c}' \\ \frac{1}{2}N_{\delta_e}' & -\frac{1}{2}N_{\delta_e}' & \frac{1}{2}N_{\delta_a}' & -\frac{1}{2}N_{\delta_a}' & N_{\delta_r}' & N_{\delta_c}' \end{bmatrix} \begin{bmatrix} \delta_1^a \\ \delta_1^b \\ \delta_2^a \\ \delta_2^b \\ \delta_r \\ \delta_c \end{bmatrix} \quad (2-3)$$

where

$\delta_1^a$  = right elevator surface

$\delta_1^b$  = left elevator surface

$\delta_2^a$  = right flaperon surface

$\delta_2^b$  = left flaperon surface

$\delta_r$  = rudder surface

$\delta_c$  = canard surface

Plant expressions are computed for each of the flight conditions, and are listed next. Due to symmetry,  $p_{ij}^a = p_{ij}^b$ .

Flight Condition 1 (Mach = 0.2 at 30 ft)

$$p_{11}^a = \frac{q}{\delta_1} = \frac{-1.118s(s+0.01822)(s+0.4568)}{(s-0.3633)(s+1.3)(s+0.07683+j0.2065)} \quad (2-4)$$

$$p_{12}^a = \frac{q}{\delta_2} = \frac{.1209s(s+0.06537)(s+0.2589)}{(s-0.3633)(s+1.3)(s+0.07683+j0.2065)} \quad (2-5)$$

$$p_{22}^a = \frac{p}{\delta_2} = \frac{-2.239s(s+0.205+j0.853)}{(s+0.1041)(s+0.6835)(s+0.2741+j1.909)} \quad (2-6)$$

$$p_{21}^a = \frac{p}{\delta_1} = \frac{-2.142s(s+0.3017+j1.562)}{(s+0.1041)(s+0.6835)(s+0.2741+j1.909)} \quad (2-7)$$

Flight Condition 2 (Mach = 0.6 at 30,000 ft)

$$p_{11}^a = \frac{-2.931s(s+0.01004)(s+0.5502)}{(s-1.167)(s+2.028)(s+0.006472+j0.07803)} \quad (2-8)$$

$$p_{12}^a = \frac{-0.1059s(s+0.006697)(s+1.861)}{(s-1.167)(s+2.028)(s+0.006472+j0.07803)} \quad (2-9)$$

$$P_{22}^a = \frac{-8.723s(s+0.219+j1.607)}{(s+0.07795)(s+0.8265)(s+0.211+j1.953)} \quad (2-10)$$

$$P_{21}^a = \frac{-6.792s(s+0.2442+j2.101)}{(s+0.07795)(s+0.8265)(s+0.211+j1.953)} \quad (2-11)$$

Flight Condition 3 (Mach = 0.9 at 20,000 ft)

$$P_{11}^a = \frac{-12.03s(s+0.01262)(s+1.51)}{(s-0.9645)(s+3.223)(s+0.007553+j0.05384)} \quad (2-12)$$

$$P_{12}^a = \frac{-3.236s(s+0.01254)(s+1.646)}{(s-0.9645)(s+3.223)(s+0.007553+j0.05384)} \quad (2-13)$$

$$P_{22}^a = \frac{-25.53s(s+0.3541+j2.927)}{(s+0.02719)(s+2.697)(s+0.391+j2.962)} \quad (2-14)$$

$$P_{21}^a = \frac{-25.36s(s+0.3749+j3.578)}{(s+0.02719)(s+2.697)(s+0.391+j2.962)} \quad (2-15)$$

Flight Condition 4 (Mach = 1.6 at 30,000 ft)

$$P_{11}^a = \frac{-16.45s(s+0.02996)(s+1.097)}{(s+0.01516+j0.02343)(s+0.8012+j6.592)} \quad (2-16)$$

$$P_{12}^a = \frac{-2.925s(s+0.03459)(s+0.6861)}{(s+0.01516+j0.02343)(s+0.8012+j6.592)} \quad (2-17)$$

$$P_{22}^a = \frac{-7.084s(s+0.4083+j4.916)}{(s+0.03448)(s+2.171)(s+0.4996+j3.129)} \quad (2-18)$$

$$P_{21}^a = \frac{-23.3s(s+0.3774+j3.848)}{(s+0.03448)(s+2.171)(s+0.4996+j3.129)} \quad (2-19)$$

### II.3 Summary

The aircraft equations of motion in state-variable format are presented in this chapter. The longitudinal and lateral axes are coupled within the control matrix allowing control of individual elevators and flaperons. Also presented are the basic plant transfer functions of interest computed from the coupled aircraft equations of motion for each of the four flight conditions.

The next chapter presents the reconfiguration system model, and quantitative feedback theory as applied to the reconfiguration.



### III. Reconfiguration Theory

#### III.1 Introduction

The concepts of QFT can be used to design fixed compensators to achieve tolerance specifications (2; 3). The specifications are to be satisfied despite highly uncertain parameters which can arise from aircraft control surface failures (partial or total), and from widely varying flight conditions. The specifications must also be satisfied for the normal system (no failures). The advantage of a fixed compensator design is that fast identification of failures is not necessary. A more optimum compensator can be switched in later, if required, after the failure state has been explicitly identified. This chapter presents the MIMO system model, and develops an equivalent plant matrix for which QFT is applicable.

#### III.2 Reconfiguration Theory and System Model

The structure of the system model is seen in Figure 3-1, and represents a four input-two output system. A two-elevator, two-flaperon system is represented with each individual surface separately controllable. The elevators and flaperons are capable of moving individually, and symmetrically or differentially in pairs. The right

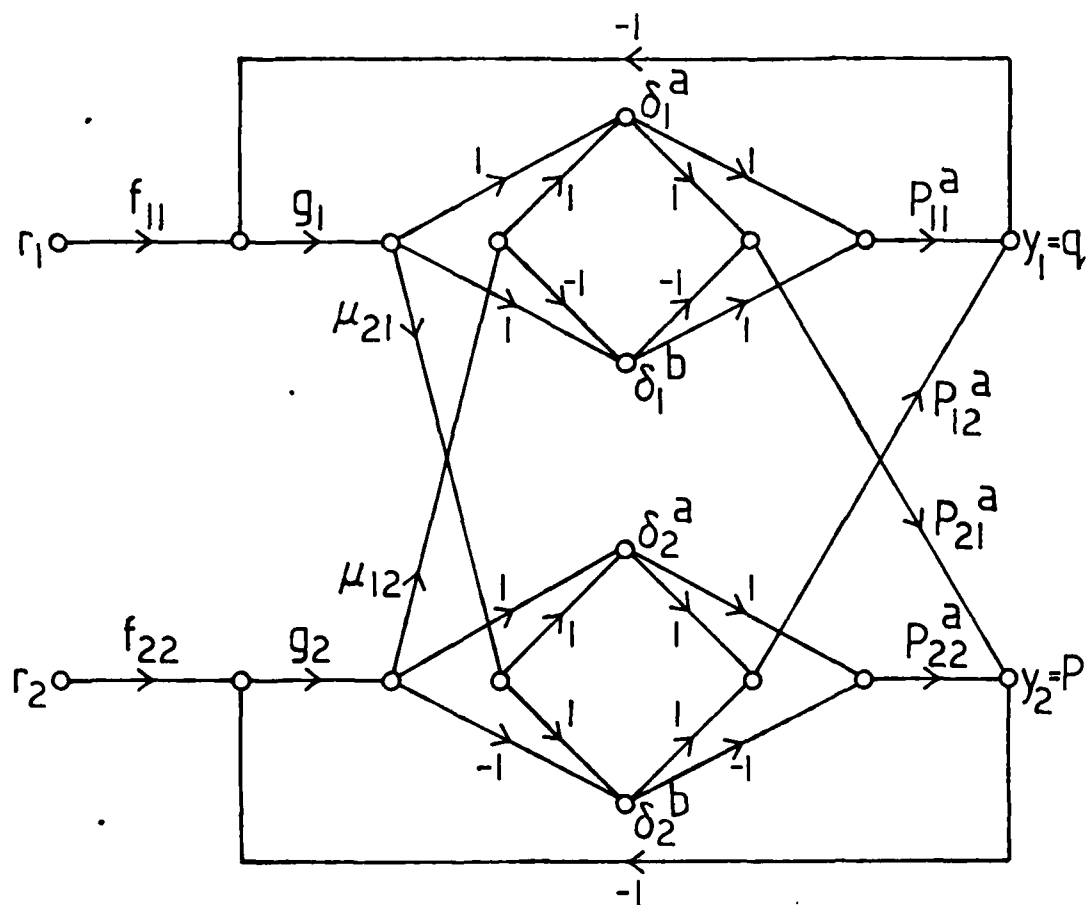


Fig. 3-1. Plant and Compensation  
Model Signal Flow Diagram

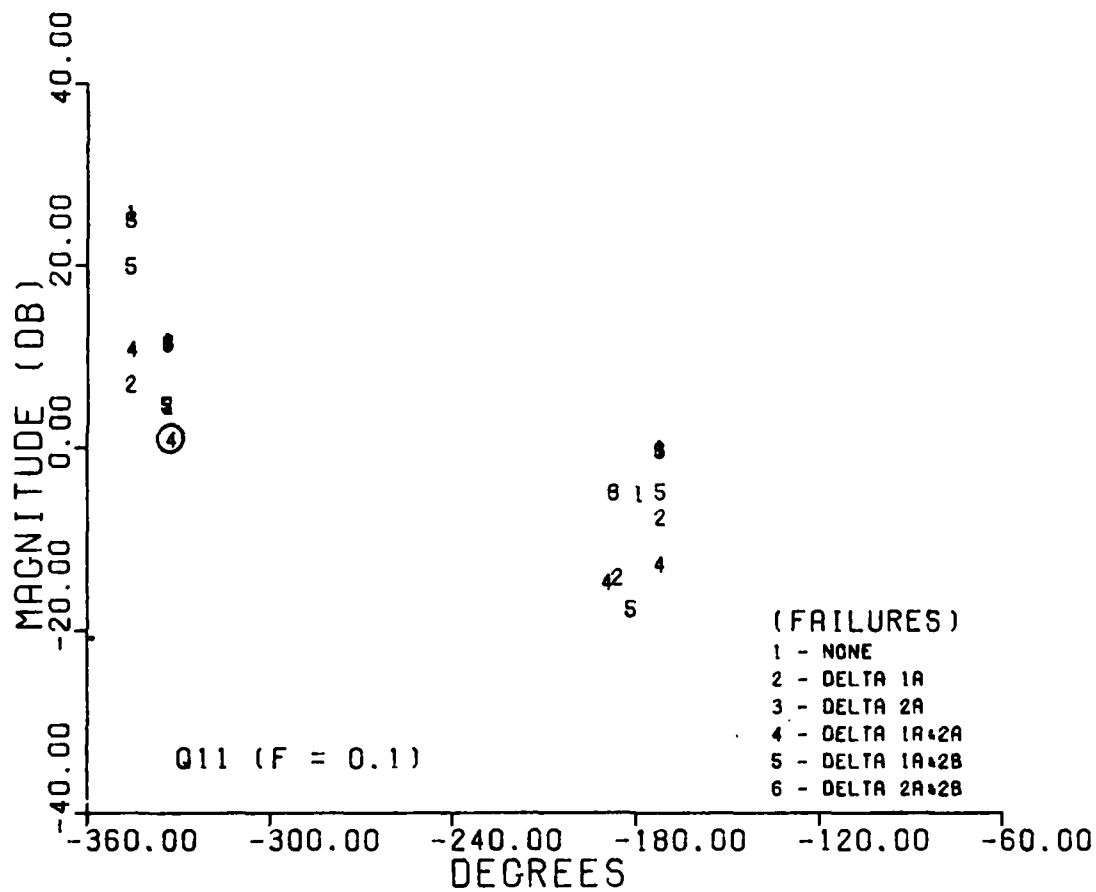


Fig. 4-6. Template of  $Q_{11}$   
at  $\omega = 0.1$  Radians/Second

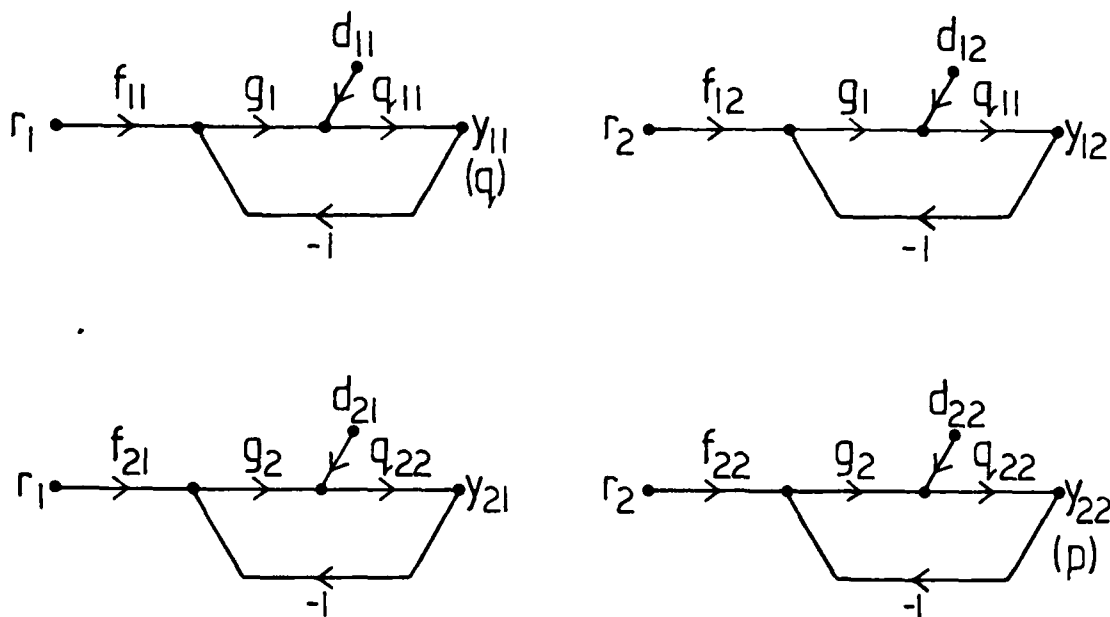


Fig. 4-5. Equivalent SISO Systems

at  $\omega = 0.1$  rad/sec are seen in Figures 4-6 and 4-7, respectively. The  $q$  channel is represented by  $Q_{11}$ , and the  $p$  channel by  $Q_{22}$ . Each template represents the variable uncertainty at a particular frequency for the six CSC modes and four flight conditions. Thus, each template has 24 points; the modes are numbered 1 through 6 as indicated on the template figures. Each CSC mode is repeated four times at a particular frequency corresponding to the four flight conditions. A nominal condition is chosen and indicated by circling that condition on each template. The nominal condition for  $Q_{11}$  is flight condition 2, CSC mode 4 (Mach = 0.6;  $\delta_1^a$  and  $\delta_2^a$  failed), while flight condition 4, CSC mode 4 (Mach = 1.6) is selected as the nominal condition

TABLE 4-1  
FIGURE OF MERIT TOLERANCES FOR PITCH RATE

Settling Time $t_s$ (sec)	Rise Time $t_r$ (sec)	Peak Value $M_p$ (deg/sec)
1.5 - 3.0	0.33 - 1.63	$\leq 11$

TABLE 4-2  
FIGURE OF MERIT TOLERANCES FOR ROLL RATE

Settling Time $t_s$ (sec)	Rise Time $t_r$ (sec)	Peak Value $M_p$ (deg/sec)
3.0 - 6.0	0.65 - 3.23	$\leq 55$

Transfer function models for roll rate are also listed in Appendix D. Once acceptable tolerances are established, bounds on the loop transmissions are formed.

#### IV.4 Loop Bound Design, $L_{20}$

The decision must be made on which loop transmission ( $L$ ) is to be designed first. Since the equivalent system is a two-by-two structure with two inputs to consider, both the command and the BNIC elements must be analyzed for each  $L$  to define the loop bounds. The equivalent SISO single-loop systems are seen in Figure 4-5.

As to which  $L$  to design first, both loops require the same analysis, so neither is less complicated than the other. For this problem, the  $L$  for the  $p$  channel is designed first because of the smaller uncertainty associated with this variable. This is seen by comparing the plant templates for  $p$  and  $q$ . Templates of  $Q_{11}$  and  $Q_{22}$

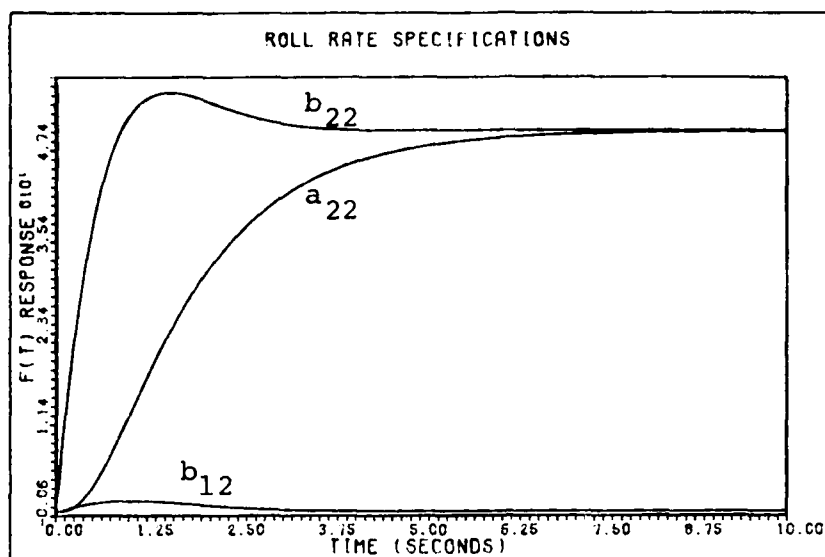


Fig. 4-3. Roll Rate Tolerances  
in the Time Domain

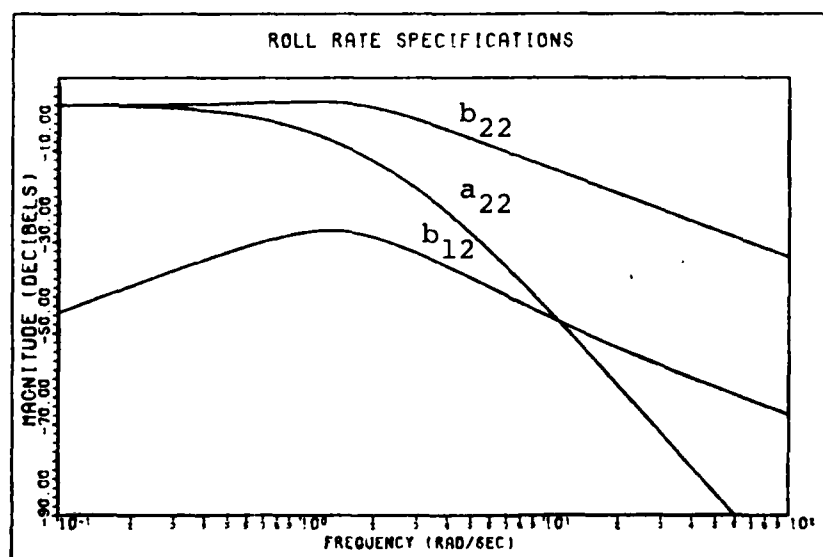


Fig. 4-4. Roll Rate Tolerances  
in the Frequency Domain

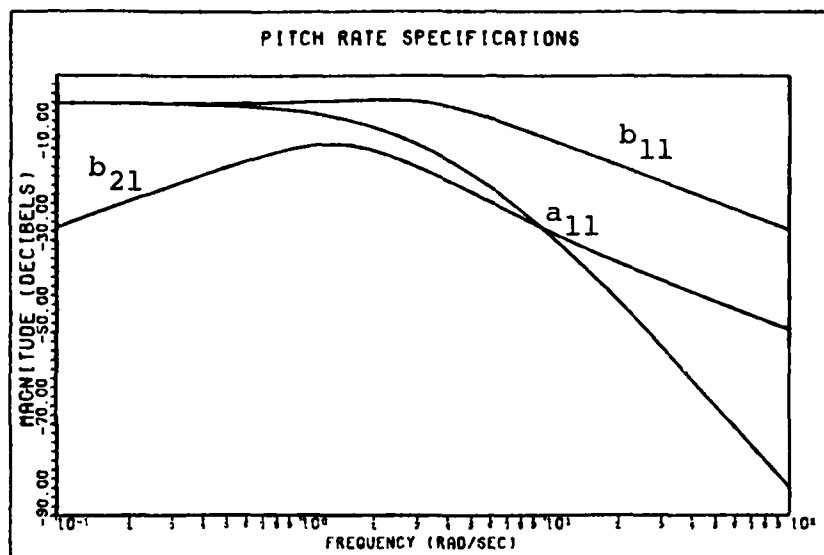


Fig. 4-2. Pitch Rate Tolerances in the Frequency Domain

merit are given in Reference 10. Tolerances for these figures of merit are seen in Table 4-1. The  $p$  tolerance  $b_{21}$  is modeled for a total roll angle of approximately five degrees. This specification is established with severe failure situations in mind, such as one elevator and one flap on failed on the same side. Five degrees of roll angle seems quite reasonable for situations such as this. Tolerances are also established for the  $p$  command responses.

Roll rate tolerances  $b_{22}$ ,  $a_{22}$ , and  $b_{12}$  are also established as is done for the  $q$  tolerances. It is desired that the  $p$  commanded be 50 deg/sec with small  $q$  resulting. The  $q$  tolerance  $b_{12}$  is modeled for a total pitch angle of approximately three degrees. Figures 4-3 and 4-4 illustrate the time and frequency domain tolerances for  $p$ , and Table 4-2 lists the figure of merit tolerances for  $p$ .

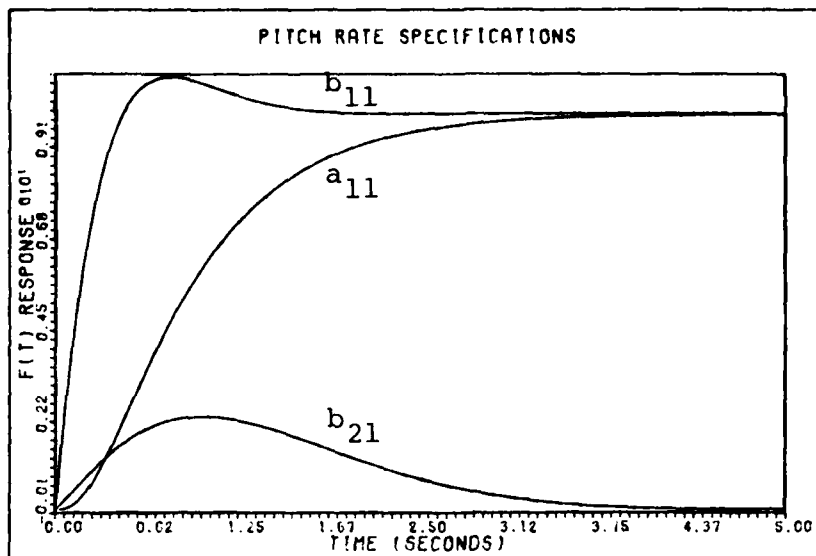


Fig. 4-1. Pitch Rate Tolerances in the Time Domain

The  $p$  response is a result of loop interaction, and is denoted by  $b_{21}$ . The time response tolerances transformed into the frequency domain are seen in Figure 4-2. As discussed earlier, all design work is accomplished in the frequency domain. Transfer function models for the response tolerances are listed in Appendix D. Figures of merit are established for the response tolerances.

Figures of merit for the outputs of interest are established primarily on what the author feels are reasonable from his personal experience gained from flying fighter aircraft. The main concern is to demonstrate the efficacy of QFT when applied to a reconfiguration problem. Settling time  $t_s$ , rise time  $t_r$ , and peak value  $M_p$  are chosen as the design figures of merit. Definitions for the figures of



realizable for that specific  $\underline{P}_e$ . The second condition, also derived in Reference 8, requires that

$$\left| \frac{Q_{11}Q_{22}}{Q_{12}Q_{21}} \right| < 1 \quad \text{as } s \rightarrow \infty \quad (4-1)$$

This must be so for all  $\underline{P}_e \in P$ , and all  $\omega > \text{some } \omega_h$ , where  $|1+L| < 1$ . This second condition is an inherent constraint. Both of the above conditions for this thesis are found to be satisfied for every  $\underline{P}_e$ , so design can proceed.

#### IV.3 Specifications

In general, a control problem exists because a set of performance criteria are specified (10). Acceptable tolerances for the closed-loop responses are established, and the goal is to find compensation which guarantees that these tolerances are satisfied over the range of plant uncertainty. In QFT it is guaranteed that the amount of feedback designed into the system is such that the desired responses are obtained despite highly uncertain plant parameters (2). Compensation is synthesized so that the specifications are satisfied for all design flight conditions and CSC modes.

The tolerances for the  $q$  and  $p$  responses of Figure 3-1, when  $q$  is commanded, are illustrated in Figure 4-1. It is desired that the  $q$  commanded be ten degrees per second (deg/sec) with small  $p$  resulting. The upper bound for the  $q$  response is denoted by  $b_{11}$  and the lower bound by  $a_{11}$ .

and CSC mode. For this study the equivalent plant set has as members the matrices for each flight condition and CSC mode, giving a total of 24 matrices (4 flight conditions times 6 CSC modes). The elements of each  $\underline{P}_e$  are listed in Appendix C. After developing the  $\underline{P}_e$ ,  $\underline{P}_e^{-1}$  is computed using the computer program Total, obtaining  $\underline{P}_e^{-1} = (1/Q_{ij})$ . The  $Q_{ij}$  become the plant elements for the equivalent SISO systems (2; 3). Appendix C also lists the individual  $Q_{ij}$  for each flight condition and CSC mode. Notice in Appendix C that some of the  $Q$ 's have unstable poles. This does not present an obstacle for this technique. This is addressed later in this chapter. Zeros in the right-half-plane, however, represent a non-minimum phase system, and, as a result, the design is more difficult. Dr. Horowitz has developed QFT to design for non-minimum phase plants (9). Since this thesis does not involve non-minimum phase plants, they are not addressed. Before proceeding with the design, certain conditions must first be satisfied for the MIMO synthesis technique to be applicable.

Two conditions must be satisfied before proceeding (8). First,  $\underline{P}_e^{-1}$  must exist in order to obtain the  $Q_{ij}$  elements for the single-loop systems. This is the "controllability" condition and must be true for each flight condition and CSC mode. If the determinant of  $\underline{P}_e$  equals zero it means that the plant outputs are not independently

## IV. Compensator Designs

### IV.1 Introduction

This chapter presents the steps used in the design of the compensators  $g_1$ ,  $g_2$  in Figure 3-1. The procedure is started by inverting the elements of the equivalent plant transfer function matrices for the various CSC modes to yield the  $Q_{ij}$  plant equations for the single-loop systems (see Figure 4-5, Section IV.4). These  $Q_{ij}$  plant equations are used in the detailed design execution. The p channel compensator is designed first due to a smaller amount of uncertainty associated with this variable. Using data derived in the design of this loop, the q channel compensator is then derived using the "improved method" (8). Simple, first-order prefilters are designed for both channels. The synthesis begins with the plant equations.

### IV.2 Plant Equations

The design process begins by obtaining the basic plant transfer function equations from Eq. (2-3) for each design flight condition. These equations are listed in Chapter II. The equivalent plant transfer function matrices ( $P_e$ ) are developed as described in Chapter III from these basic plant equations. Equivalent plant transfer function matrices are developed for each flight condition

#### III.4 Summary

Presented in this chapter is the multiple input-multiple output system model and reconfiguration theory. An equivalent square plant matrix is also developed from a non-square plant matrix so that quantitative feedback theory QFT can be applied. In the next chapter, the compensator design procedures and the results are presented.

Substitutions are made into Eq. (3-7) for the various control surface configurations producing closed-loop transfer functions for each command and cross coupling response. For example, consider the case in which the right elevator has failed ( $\delta_1^a = 0$ ); this implies that  $P_{11}^a$  and  $P_{21}^a$  are equal to zero. With these substitutions, the equivalent plant matrix becomes

$$\underline{P}_e = \begin{bmatrix} P_{11}^b + \mu P_{12} & \Delta P_{12} + \mu P_{11}^b \\ -P_{21}^b + \mu \Delta P_{22} & P_{22} + \mu P_{21}^b \end{bmatrix} \quad (3-8)$$

This new equivalent plant matrix is used to determine the four  $t_{ij}$  transfer functions. Each control surface failure situation can be treated in this manner to obtain the resulting plant transfer function. The various failures give different equivalent plant matrices for each flight condition. Quantitative feedback theory is applied to the entire set. The number of elements in the set  $\underline{P}_e$  is the product of the number of CSC modes considered by the number of flight conditions. Appendix C lists the entire  $\underline{P}_e$  set used in this thesis. QFT is well suited for this type of problem since the surface failures merely enlarge the plant set. The extra "cost of feedback" can readily be seen by the designer.

$$\underline{\delta} = \begin{bmatrix} f_1 g_1 & \mu f_2 g_2 \\ f_1 g_1 & -\mu f_2 g_2 \\ \mu f_1 g_1 & f_2 g_2 \\ \mu f_1 g_1 & -f_2 g_2 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} - \begin{bmatrix} g_1 & \mu g_2 \\ g_1 & -\mu g_2 \\ \mu g_1 & g_2 \\ \mu g_1 & -g_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (3-5)$$

From Eqs. (3-1) to (3-5), and by letting  $P_{ij} = P_{ij}^a + P_{ij}^b$ , and  $\Delta P_{ij} = P_{ij}^a - P_{ij}^b$ , an equivalent plant matrix  $P_e$ , emerges as follows:

$$\underline{P}_e = \begin{bmatrix} P_{11} + \mu P_{12} & \Delta P_{12} + \mu \Delta P_{11} \\ \Delta P_{21} + \mu \Delta P_{22} & P_{22} + \mu P_{21} \end{bmatrix} = \begin{bmatrix} P_{11e} & P_{12e} \\ P_{21e} & P_{22e} \end{bmatrix} \quad (3-6)$$

Now the plant matrix is square, and QFT is directly applicable. This two-by-two MIMO system can be reduced to four SISO single-loop systems for the design synthesis (2; 3).

After completing all the matrix operations the final expression for  $\underline{T}$  becomes

$$\underline{T} = \{t_{ij}\}$$

$$= \frac{\begin{bmatrix} \{f_1 g_1 P_{11e} (g_2 P_{22e} + 1) - f_1 g_1 g_2 P_{12e} P_{21e}\} & \{f_2 g_2 P_{11e}\} \\ \{f_1 g_1 P_{21e}\} & \{f_2 g_2 P_{22e} (g_1 P_{11e} + 1) - f_2 g_1 g_2 P_{12e} P_{21e}\} \end{bmatrix}}{(g_1 P_{11e} + 1)(g_2 P_{22e} + 1) - f_1 f_2 g_1 g_2 P_{12e} P_{21e}} \quad (3-7)$$

failure. In Figure 3-1,  $\mu_{12}$  and  $\mu_{21}$  are constants which determine the division of control effort between the flaperons and elevators. For this preliminary study, optimization of these constants is not addressed, and both  $\mu$ 's are set equal to 0.25 for the majority of the simulations. Henceforth, the subscripts are dropped on these two constants. Preliminary theory has been developed to enable the designer to specify the optimum division of control effort. Appendix B contains this work along with a detailed explanation of reconfiguration theory for larger, more complex systems. The next section presents the development of an equivalent plant for which QFT is applicable.

### III.3 Equivalent Plant Development

$$\text{In Figure 3-1, } \underline{Y} = \underline{P}\delta = \underline{P}\{\underline{G}\underline{F}\underline{r} - \underline{G}\underline{Y}\} \quad (3-1)$$

Thus,

$$\underline{T} = \{t_{ij}\} = \{\underline{I} + \underline{P}\underline{G}\}^{-1} \underline{P}\underline{G}\underline{F} \quad (3-2)$$

where  $t_{ij} = (\underline{I} + \underline{P}\underline{G})^{-1}$ .

$$\underline{P} = \begin{bmatrix} P_{11}^a & P_{11}^b & P_{12}^a & P_{12}^b \\ P_{21}^a & -P_{21}^b & P_{22}^a & -P_{22}^b \end{bmatrix} \quad (3-3)$$

$$\underline{\delta} = [\delta_1^a, \delta_1^b, \delta_2^a, \delta_2^b] \quad (3-4)$$

and left elevators are designated by  $\delta_1^a$  and  $\delta_1^b$ , respectively; the flaperons are designated similarly by  $\delta_2^a$  and  $\delta_2^b$ . The longitudinally controlled output variable is pitch rate ( $q$ ), while the laterally controlled output variable is roll rate ( $p$ ). As stated in Chapter II, the plant transfer function designator,  $P_{ij}$ , relates the  $i$ th output to the  $j$ th input (control surface). A superscript "a" or "b" denotes the right or left control surface. For example,  $P_{21}^a$  denotes the effect of the right elevator ( $\delta_1^a$ ) on roll rate (output  $y_2$ ). For a normally operating system,  $P_{ij}^a = P_{ij}^b$ . Minus signs are associated with  $P_{21}$  and  $P_{22}$  so that the two pairs of surfaces move differentially to generate roll rate. In the no fail case, surfaces move an equal amount in either direction to generate the commanded rates. No interaction is desired between the longitudinal and lateral modes under normal operating conditions.

Normally, the elevators move together in the same direction to excite only the longitudinal mode, and the flaperons move opposite each other to excite only the lateral mode. Separate control of the surfaces increases the flexibility of a system by exciting both aerodynamic modes when a single surface is actuated. For instance, using the structure of Figure 3-1, the surfaces can continue to act in pairs under normal operation, but act separately, as necessary, to offset parameter changes resulting from single or multiple control system component



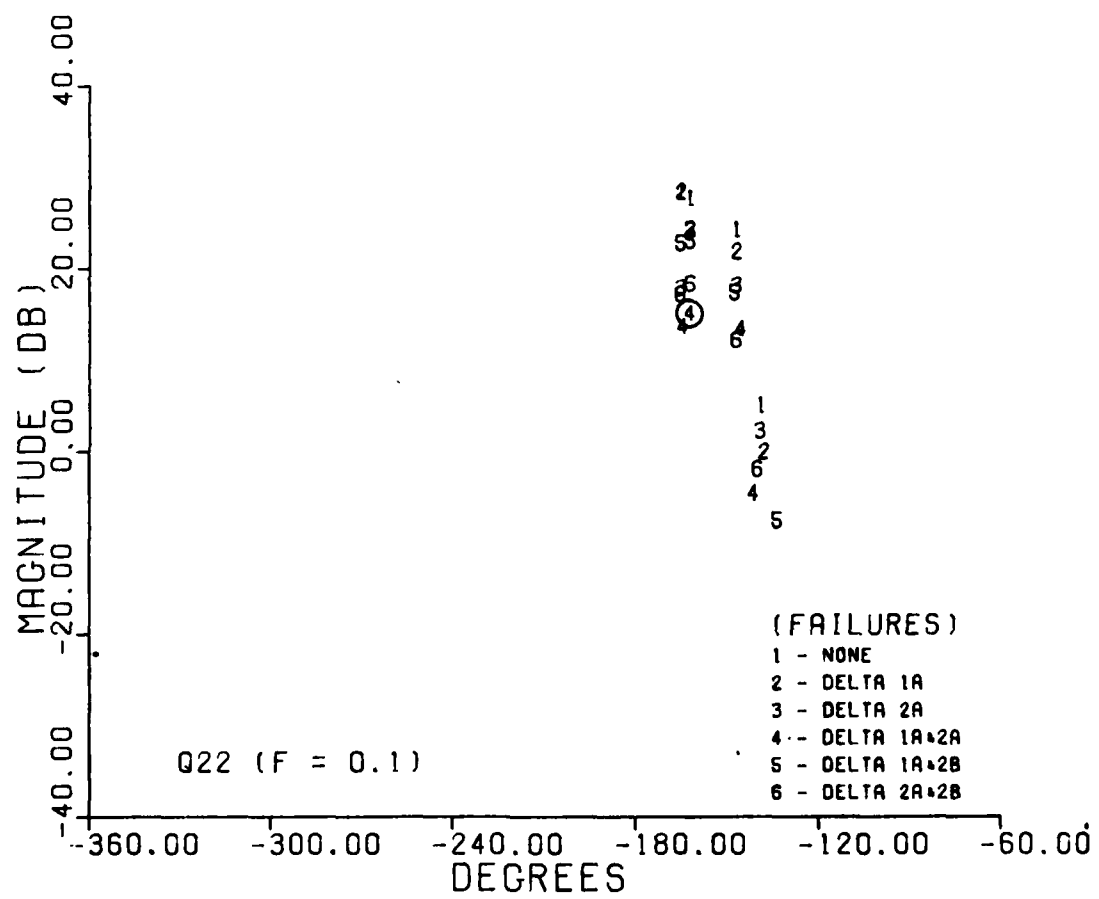


Fig. 4-7. Template of  $Q_{22}$   
at  $\omega = 0.1$  Radians/Second

for  $Q_{22}$ . Dr. Horowitz recommends using the same nominal condition for each loop design. This eliminates any confusion as to the nominal condition when solving for the compensator explicitly. Templates are made for frequencies approximately one octave apart until the locus of points is essentially a vertical line. Appendix E contains templates of  $Q_{11}$  and  $Q_{22}$  at several frequencies. An analysis of the p channel is accomplished next.

The BNIC system whose output is  $y_{21}$  is analyzed first since only the input due to loop interaction is considered. A response due to  $r_1$  does not exist for this single-loop system since the prefilter  $f_{21}$  is chosen to be zero. Consider the expression for  $y_{21}$

$$y_{21} = \frac{d_{21}Q_{21}}{1+g_2Q_{22}} \quad (4-2)$$

where

$$d_{21} = t_{11}/Q_{21}(2), \text{ and } L_2 = g_2Q_{22} \quad (4-3a,b)$$

Rewriting Eq. (4-2)

$$y_{21} = \frac{(t_{11}/Q_{21})Q_{22}}{1+L_2} \leq b_{21} \quad (4-4)$$

with  $b_{21}$  being the specification bound for p due to the q command. To ensure total compliance for all acceptable  $t_{11}$ , the upper bound of  $t_{11}$  for the q response becomes  $b_{11}$ . Upon rearranging terms, Eq. (4-4) becomes

$$\left| \frac{1}{1+L_2} \right| \leq \left| \frac{b_{21}Q_{21}}{b_{11}Q_{22}} \right| \quad (4-5)$$

In order to determine the most demanding bound, use the minimum value of  $Q_{21}$  for all flight conditions and CSC modes at the frequency in question. Also, use the nominal value for  $Q_{22}$ ,  $Q_{220}$ . This procedure results in overdesign unless  $|L_2| \gg 1$ . If  $|L_2| \gg 1$  is not a valid assumption, then  $Q_{21}$  and  $Q_{22}$  in Eq. (4-5) should correspond to the same flight condition and CSC mode. This takes into account the correlation between  $Q_{21}$  and  $Q_{22}$ . The bound on  $L_{20}$  is then formed from the most demanding of all the bounds from Eq. (4-5) at each phase angle. For this particular problem, the demands on  $L_{20}$  are high in the low frequency region making  $|L_2| \gg 1$  a valid assumption. Equation (4-6) is used to locate bounds on  $L_{20}$  up to  $\omega = 2$  r/s. At higher frequencies another technique, described later, is used. Equation (4-5) now becomes

$$\left| \frac{1}{1+L_{20}} \right| \leq \left| \frac{b_{21}Q_{21_{\min}}}{b_{11}Q_{220}} \right| \quad (4-6)$$

The resulting bound is the bound for the nominal  $L_2$ ,  $L_{20}$ . The terms on the right hand side of Eq. (4-6) are constants obtained from the Bode plots of their transfer functions. In the region where  $|1+L_{20}| \approx |L_{20}|$ , Eq. (4-6) becomes

$$|L_{20}| \geq \left| \frac{b_{11}Q_{220}}{b_{21}Q_{21\min}} \right| \quad (4-7)$$

and the demand on  $L_{20}$  is obtained explicitly. Equation (4-7) results in a contour on the modified Nichol's Chart (MNC), also referred to as the inverted Nichol's Chart, and is compared with an analysis of the  $y_{22}$  loop of Figure 4-5 to produce the most demanding bound on  $L_{20}$ . A detailed explanation of the MNC is presented in Reference 2. Using Eq. (4-7), at  $\omega = 0.1$  r/s  $L_{20}$  is calculated as follows:

$$|L_{20}| \geq 0.172+27-(-26.9-22.5) \approx 76 \text{ decibels (dB)}$$

Parallel vertical lines are used to indicate the log magnitude (LM) operation. The bound on  $L_{20}$  at  $\omega = 0.1$  r/s coincides with the horizontal line for  $L_{20}$  because the  $(1+L_{20})^{-1}$  contour at -76 dB is also a horizontal line.

The  $y_{22}$  loop in Figure 4-5 has two inputs to consider,  $r_2$  and  $d_{22}$ , and therefore two transfer function expressions must be analyzed. The expression for  $t_{22}$  is

$$t_{22} = \tau_{22} + \tau_{d_{22}} \quad (4-8)$$

where

$$\tau_{22} = \frac{f_{22}q_2Q_{22}}{1+q_2Q_{22}} = \frac{f_{22}L_2}{1+L_2} \quad (4-9)$$

$$\tau_{d_{22}} = \frac{d_{22}Q_{22}}{1+L_2} = \frac{(b_{12}/Q_{21})Q_{22}}{1+L_2} \quad (4-10)$$

Using the value of  $L_{20}$  found in the analysis of  $y_{21}$ ,  $\tau_{d_{22_{\max}}}$  is determined from the expression

$$|\tau_{d_{22_{\max}}}| = \left| \frac{b_{12}Q_{220}}{L_{20}Q_{21_{\min}}} \right| \quad (4-11)$$

Next, the template of  $q_{22}$  for the frequency in question is taken and placed on the Nichol's Chart to determine

$|\tau_{22_{\max}}/\tau_{22_{\min}}|$ . If Eq. (4-12), below, is satisfied, then the bound found in the first analysis satisfies both loops and is used as the bound on  $L_{20}$  for that one frequency.

$$|2\tau_{d_{22_{\max}}}| + |\tau_{22_{\max}}| - |\tau_{22_{\min}}| \leq \left| \frac{(b_{22} - a_{22})}{2} \right| \quad (4-12)$$

If Eq. (4-12) is not satisfied, then the template of  $Q_{22}$  is translated vertically until it is satisfied. The contour through the  $Q_{220}$  point becomes the new bound on  $L_{20}$ . For this thesis it is found that the demands from  $y_{21}$  are the most demanding and, as a result, determine the bounds on  $L_{20}$  for  $\omega_i$  such that  $|1+L_{20}| \approx |L_{20}|$ . At higher frequencies this approximation is not valid, and another method is used to locate the bounds on  $L_{20}$ .

The lower specification bound  $a_{22}$  is less than -12 dB for  $\omega > 2$  r/s. The  $LMf_{22}$  is also relatively small in the region since it is sloping down at approximately the same rate as  $a_{22}$ . In Eq. (4-8) the small magnitude of  $f_{22}$  makes  $\tau_{22}$  negligible in comparison to  $\tau_{d_{22}}$ . This

leaves  $\tau_{d_{22}}$  and  $\tau_{d_{21}}$  to determine the bounds on  $L_{20}$  for  $\omega > 2$  r/s. It is found that the -4 dB contour satisfies the constraints imposed by both  $\tau_{d_{22}}$  and  $\tau_{d_{21}}$  in this region. This greatly simplifies the task of determining bounds. The template of  $Q_{22}$  is translated down and horizontally as necessary on the Nichol's Chart so that the template remains outside, but as close as possible to the -4 dB contour. The trace of the nominal point during this procedure becomes the bound on  $L_{20}$ . This procedure is repeated for several frequencies until the universal high frequency bound is formed (2). Figure 4-8 shows  $L_{20}$  and its bounds plotted on the Nichol's Chart, and the transfer function expression for  $L_{20}$  is

$$L_{20} = \frac{5.95(10^6)(s+0.3)(s+12.9)(s+120)}{s(s+0.2)(s+3.46)(s+48)(s^2+900s+562,500)} \quad (4-13)$$

Inspection of Figure 4-8 shows that  $L_{20}$  is close to its bounds for the lower frequencies, and avoids the -4 dB contours at high frequency. If  $L_{20}$  were optimally designed, it would lie exactly on its bound at each frequency, and the loop transmission bandwidth would be minimum. The bandwidth of the loop transmission is an important consideration, and if too large, the system can be susceptible to noise. Using Total, the bandwidth of  $L_{20}$  is found to be 34 r/s. No attempt to analyze the system for susceptibility

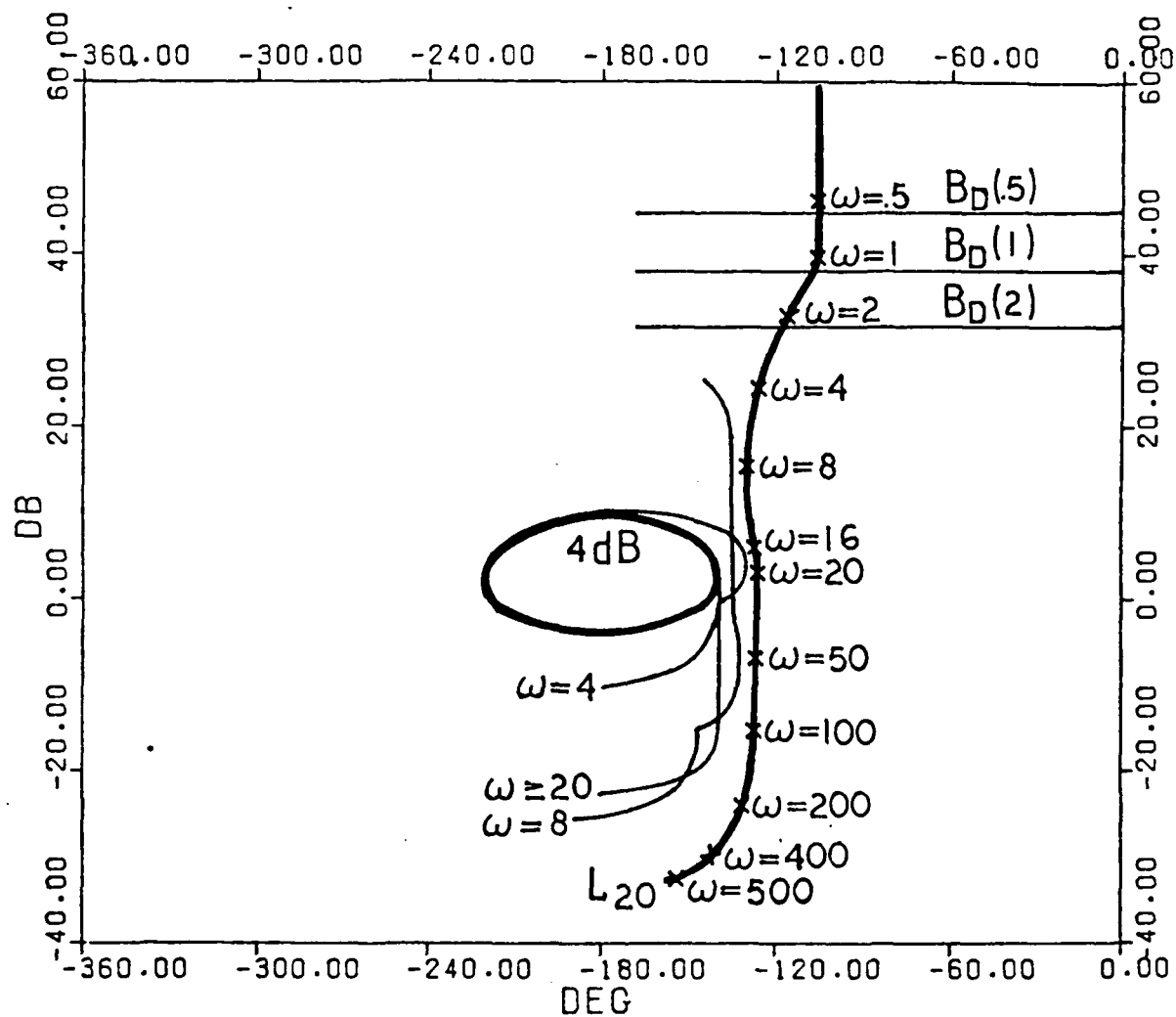


Fig. 4-8. Nichol's Chart Plot  
of  $L_{20}$  and Bounds

to noise is made in this thesis. The Bode plot of  $L_{20}$  is seen in Figure 4-9, and the compensator  $g_2$  is derived from  $L_{20}$ .

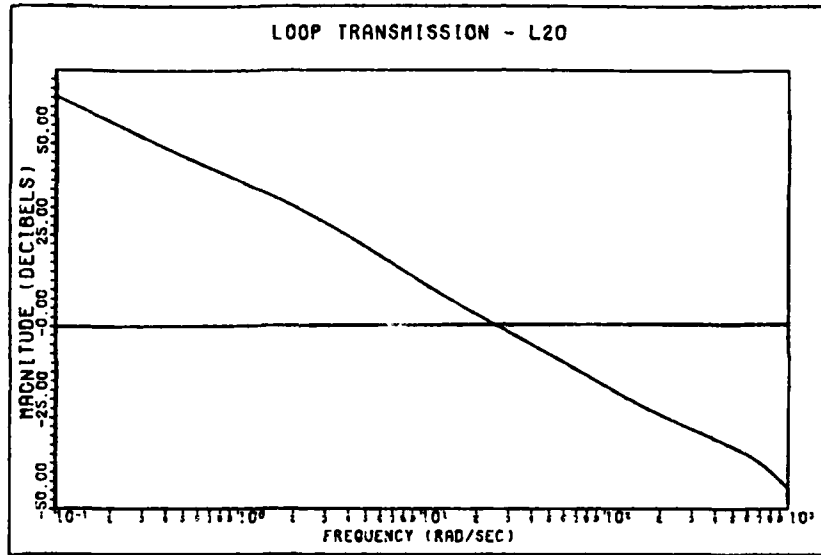


Fig. 4-9. Bode Plot of  $L_{20}$

The compensator  $g_2$  is obtained from the following equation

$$g_2 = L_{20}/Q_{220} \quad (4-14)$$

where

$$Q_{220} = \left[ \frac{-2.64s(s^2 + 1.2412s + 37.33)}{(s + 0.03448)(s + 2.171)(s^2 + 0.9992s + 10.04)} \right] \quad (4-15)$$

and

$$g_2 = \left[ \frac{-2.25(10^6)(s + 0.03448)(s + 0.3)(s + 2.171)(s + 12.9)(s + 120)(s^2 + 0.9992s + 10.04)}{s^2(s + 0.2)(s + 3.46)(s + 48)(s^2 + 1.24s + 37.3)(s^2 + 900s + 562,500)} \right] \quad (4-16)$$



After  $L_{20}$  is determined, the prefilter  $f_{22}$  is derived.

Using the methods of Reference 2,  $f_{22}$  is derived to be

$$f_{22} = \frac{2.78}{(s^2 + 2.668s + 2.78)} \quad (4-17)$$

Due to a limitation on the number of equations that the simulation program can accommodate,  $f_{22}$  is simplified to

$$f_{22} = \frac{2}{(s+2)} \quad (4-18)$$

Equation (4-18) is a good approximation of Eq. (4-17) in the important low frequency region, and satisfies the prefilter design criteria of Reference 2. The compensator  $g_2$  is not guaranteed to be without some overdesign, but a method to reduce some of the overdesign has been developed. This method is known as the "improved method," and is applied in the design of compensator  $g_1$  in Section IV.5 (8).

#### IV.5 Loop Bound Design, $L_{10}$

The exact equations for  $t_{22}$  and  $t_{21}$  are substituted into  $t_{11}$  and  $t_{12}$  to yield the following equations

$$t_{11} = \frac{f_{11}L_{1e}}{1+L_{1e}} \quad (4-19)$$

where

$$L_{1e} = q_1 Q_{11e} = \frac{q_1 Q_{11} (1+L_2)}{(1-\gamma_{12}+L_2)} \quad (4-20a)$$

and

$$\gamma = \frac{Q_{11}Q_{22}}{Q_{12}Q_{21}} \quad (4-20b)$$

It is found that in this problem  $\gamma \ll 1$  for all cases.

Simplifying Eq. (4-20a) to  $L_{1e} = L_1$ , then Eq. (4-19) becomes

$$t_{11} = \frac{f_{11}L_1}{L+L_1} \quad (4-21)$$

The  $y_{12}$  equation becomes

$$t_{12} = \frac{f_{22}L_2}{Q_{12}\{(1+L_1)(1+L_2)-\gamma\}} \quad (4-22)$$

For  $\omega_i$  such that  $1+L_2 \approx L_2$ , Eq. (4-22) reduces to

$$t_{12} = \frac{f_{22}}{Q_{12}(1+L_1)} \quad (4-23)$$

Equations (4-21) and (4-23) are used to derive the bounds on  $L_{10}$ . The procedure is similar to that used for  $L_{20}$ , except  $t_{11}$  consists only of the term due to the command input. The loop interaction term disappears since this is a BNIC problem. In the low frequency region where  $|1+L_1| \approx |L_1|$ , the  $t_{12}$  expression creates the greatest demands on  $L_{10}$ , and the following inequality is used to derive the bounds in this region.

$$t_{12} = \frac{f_{22}}{Q_{12}(1+L_1)} \leq b_{12} \quad (4-24)$$

$$\left| \frac{1}{1+L_1} \right| \leq \left| \frac{b_{12}Q_{12}}{f_{22}} \right| \quad (4-25)$$

for  $|1+L_1| \approx |L_1|$

$$|L_{10}| \geq \left| \frac{f_{22}}{b_{12}Q_{12_{\min}}} \right| \quad (4-26)$$

As an example, at  $\omega = 0.1$  r/s, using Eq. (4-26)  $L_{10}$  is calculated to be

$$|L_{10}| \geq 0 - (-45.5 + 1.34) \approx 44 \text{ dB}$$

This becomes the bound for  $L_{10}$  at  $\omega = 0.1$  r/s since the contour of  $(1+L_{10})^{-1}$  is essentially a horizontal line on the MNC for this value of  $L_{10}$ . Bounds in the medium and high frequency regions are obtained as is done for the  $L_{20}$  bounds in the same regions. The loop transmission  $L_{10}$  is shaped next.

The nominal  $q_{11}$  selected for  $L_{10}$  is

$$q_{110} = \frac{-2.235s(s+.0107)(s+.4845)}{(s-1.167)(s+2.028)(s^2+.012944s+.006131)} \quad (4-27)$$

Notice that Eq. (4-27) has an unstable pole at  $s = 1.167$ . To avoid trying to achieve exact cancellation,  $L_{10}$  must be shaped with this unstable pole as a factor. One technique is to shape  $L_{10}$  to include the pole and zero combination,  $\frac{(s+1.167)}{(s-1.167)}$ . This expression has a magnitude of one, and

a phase angle that decreases with increasing frequency.

Each bound on  $L_{10}$  as derived above is shifted by the

$\arg \left[ \frac{s+1.167}{s-1.167} \right]$ , and  $L_{10}'$  is shaped using the new bounds.

The desired  $L_{10}$  is then obtained by shifting  $L_{10}'$  back in the direction of the original bounds by the same number of degrees. The relationship between  $L_{10}$  and  $L_{10}'$  is

$$L_{10} = L_{10}' \frac{(s+1.167)}{(s-1.167)} \quad (4-28)$$

For ease of loop shaping, this method is found to be preferable to including only the unstable pole. The Nichol's Chart plots of  $L_{10}$  and  $L_{10}'$  are seen in Figure 4-10, and the Bode plot of  $L_{10}$  is seen in Figure 4-11. Figure 4-10 shows that there is overdesign in  $L_{10}$ . The bandwidth of  $L_{10}$  is found to be 24 r/s. Dividing  $L_{10}$  by  $Q_{110}$ ,  $g_1$  is found to be

$$g_1 = \frac{-2.39(10^6)(s+.3)(s+1.1)(s+1.167)(s+2.028)(s+20)(s+430)(s^2+.012944s+.00613)}{s^2(s+.0107)(s+.4845)(s+.5)^2(s+35)(s+87)(s^2+1272s+1,123,600)} \quad (4-29)$$

Notice that  $(s-1.167)$  does not appear as a factor in the denominator of  $g_1$ , and exact cancellation is not a problem.

A simple design is found to suffice for  $f_{11}$ . The expression for  $f_{11}$  is

$$f_{11} = 2/(s+2) \quad (4-30)$$

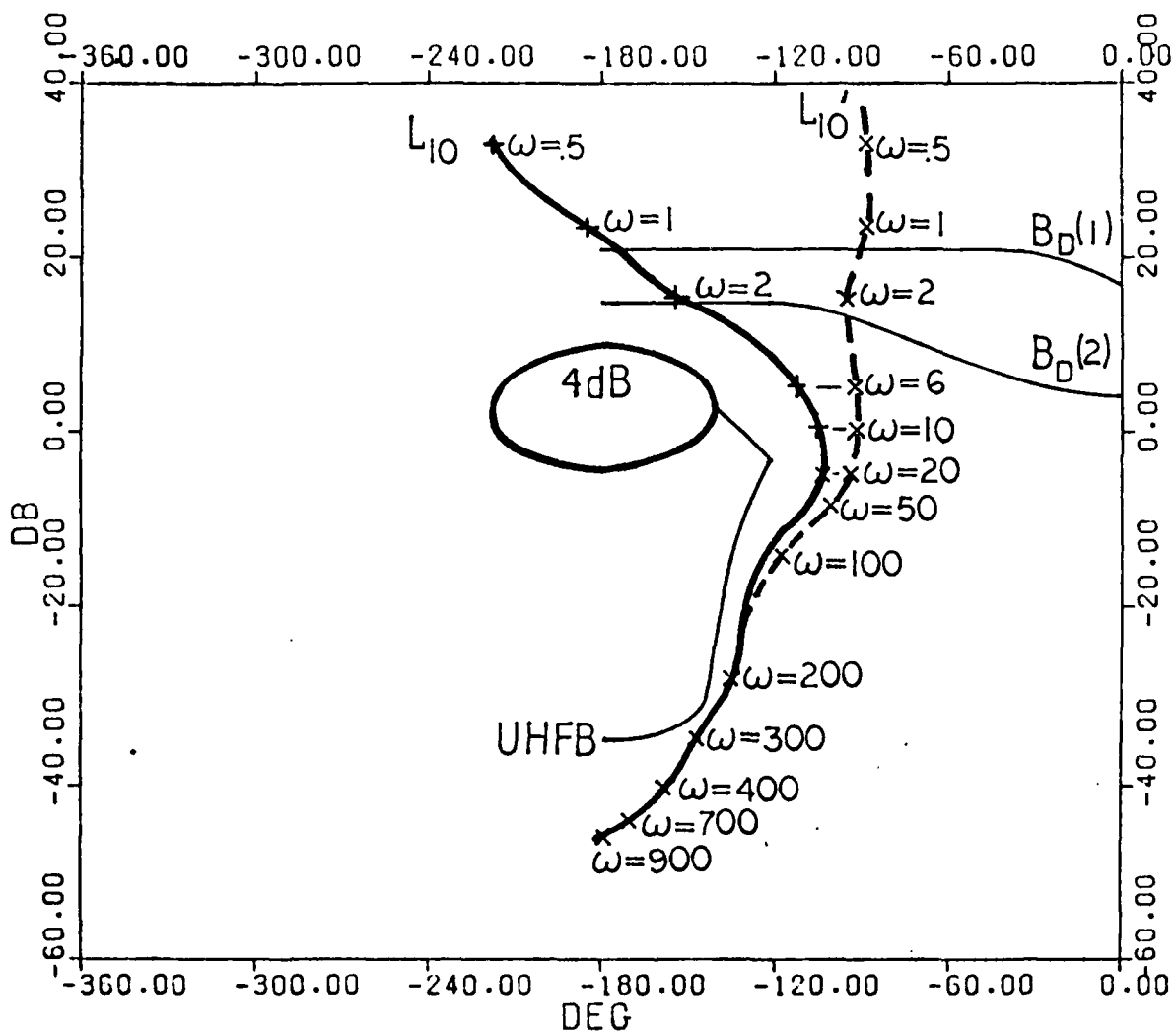


Fig. 4-10. Nichol's Chart Plot of  $L_{10}$ ,  $L_{10}'$ , and Bounds

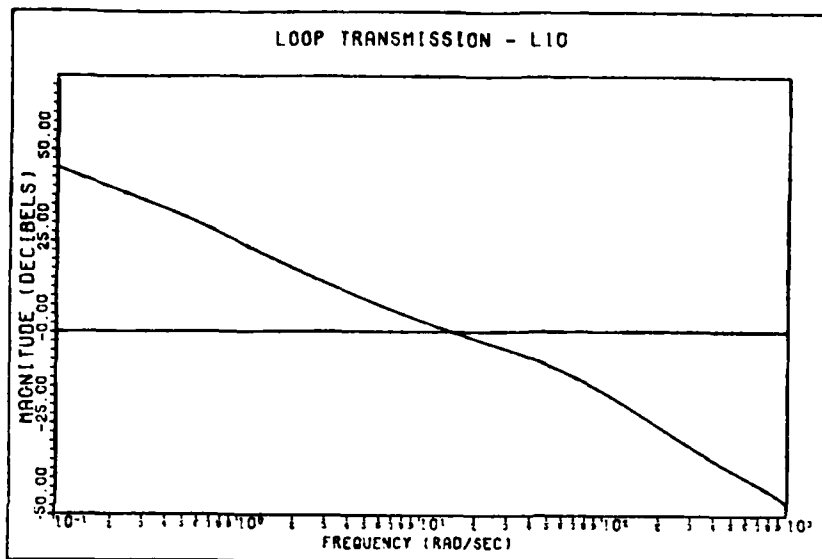


Fig. 4-11. Bode Plot of  $L_{10}$

#### IV.6 Stability Design for Three Simultaneous Failures

In this section the characteristic polynomials are derived for two configurations with three of the four control surfaces failed. For this study, it is desired to achieve only a stable system. No attempt is made to meet performance specifications. Bounds for stability are formed using templates that include all six CSC modes previously analyzed plus the configuration of interest for which only one surface is controllable. As before, a nominal condition is chosen and the stability bounds are drawn on the Nichol's Chart. The 3 dB contour is used for gain and phase margins. The nominal loop transmission is then shaped as is done for  $L_{20}$  in Chapter IV. By examining the zeros of the characteristic polynomial, which is a function

TABLE 5-2

## PITCH RATE RESPONSE FIGURES OF MERIT

Flight Condition	CSC Mode	Settling Time $t_s$ (sec)	Rise Time $t_r$ (sec)	Peak Value $M_p$ (deg/sec)
1	1	2.16	1.15	9.96
	2	3.12	1.22	9.91
	3	2.20	1.14	9.97
	4	3.24	1.22	9.90
	5	3.84	1.28	9.91
	6	2.16	1.14	9.96
2	1	2.84	1.30	10.40
	2	3.60	1.50	10.80
	3	2.80	1.25	10.40
	4	3.68	1.75	11.00
	5	3.54	1.50	10.80
	6	2.80	1.29	10.40
3	1	1.90	1.10	10.00
	2	1.87	1.10	10.10
	3	1.90	1.10	10.00
	4	2.22	1.19	10.20
	5	1.88	1.10	10.10
	6	1.90	1.10	10.00
4	1	1.86	1.07	9.87
	2	1.70	1.05	9.67
	3	1.88	1.00	9.87
	4	1.64	0.98	9.36
	5	1.72	1.03	9.68
	6	1.86	1.06	9.86

compliance with the performance tolerances listed in Table 4-1, repeated here as Table 5-1. Settling time, rise time, and peak value are calculated for each response and listed in Table 5-2.

TABLE 5-1  
FIGURE OF MERIT TOLERANCES  
FOR PITCH RATE

Settling Time $t_s$ (sec)	Rise Time $t_r$ (sec)	Peak Value $M_p$ (deg/sec)
1.5 - 3.0	0.33 - 1.63	$\leq 11$

From Table 5-2 it is seen that design tolerances for  $t_s$  are not satisfied in all instances. The  $t_s$  calculated for modes 2, 4, and 5 of flight conditions 1 and 2 is found to exceed the maximum allowed  $t_s$  of 3 seconds. An explanation for the increased settling times at flight conditions 1 and 2 may lie in the relative locations of the poles and zeros of the basic plant equations  $P_{11}^a$  and  $P_{12}^a$ , Sec. II.2. The poles of these basic plant equations for  $q$  at flight conditions 3 and 4 tend to be located farther to the left in relation to their zeros than are corresponding poles of the same plant equations for flight conditions 1 and 2. Therefore, system closed-loop roots are probably more negative at flight conditions 3 and 4 and the resulting settling times will be greater at flight conditions 1 and 2. The  $t_s$  is easily reduced by decreasing the



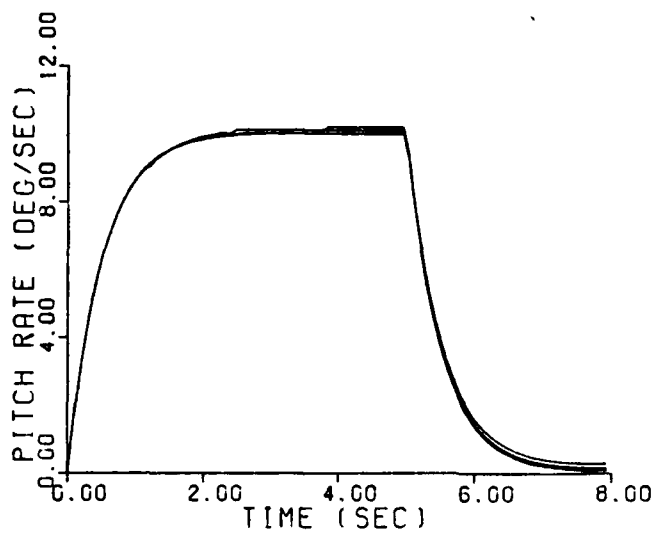


Fig. 5-2(c). Pitch Rate Responses,  
Flight Condition Three

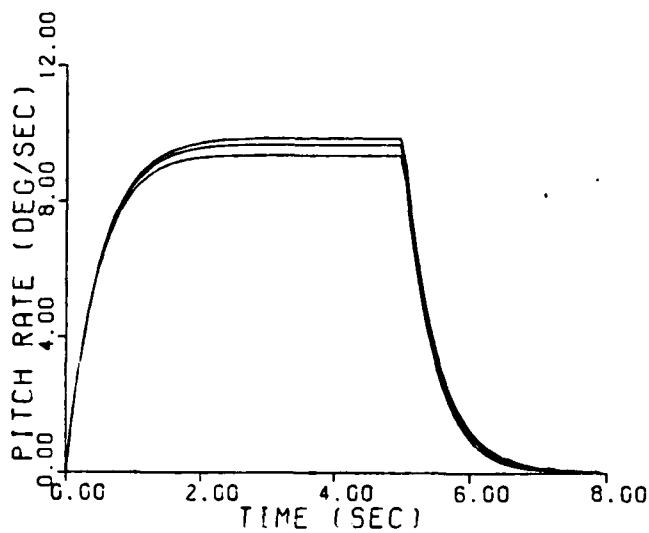


Fig. 5-2(d). Pitch Rate Responses,  
Flight Condition Four

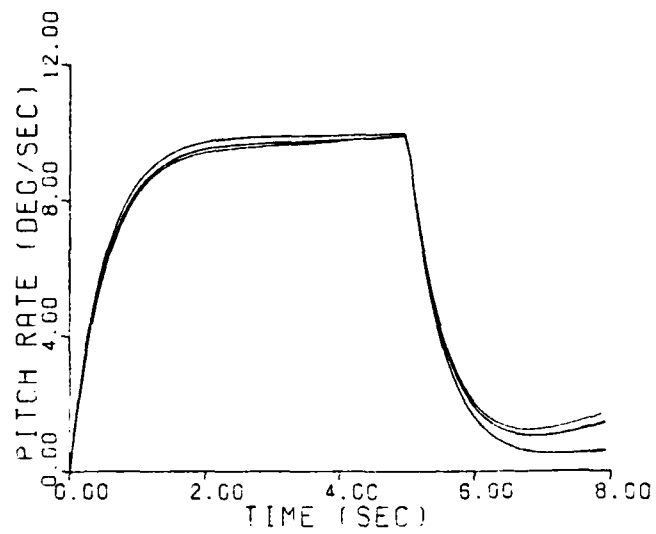


Fig. 5-2(a). Pitch Rate Responses,  
Flight Condition One

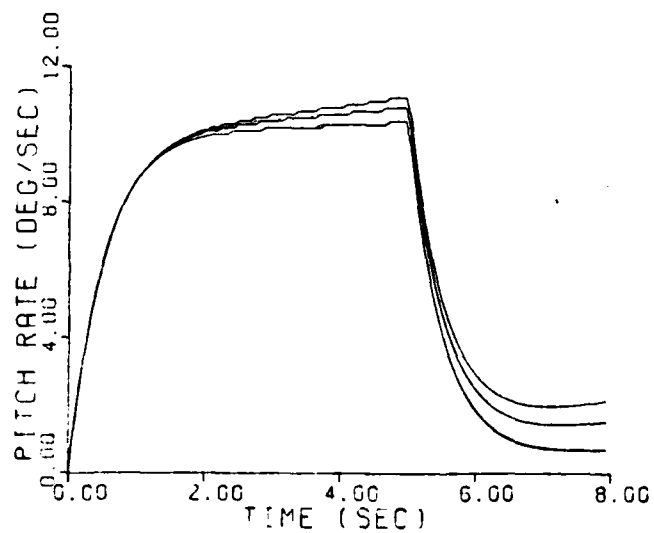


Fig. 5-2(b). Pitch Rate Responses,  
Flight Condition Two

is necessary due to a limitation on the number of equations that the simulation program can accommodate. Figure 5-1 shows a Bode plot comparison of Eqs. (4-28) and (5-1). Reducing the order to  $q_1$  by one-half is certain to make the implementation of the controller more practical.

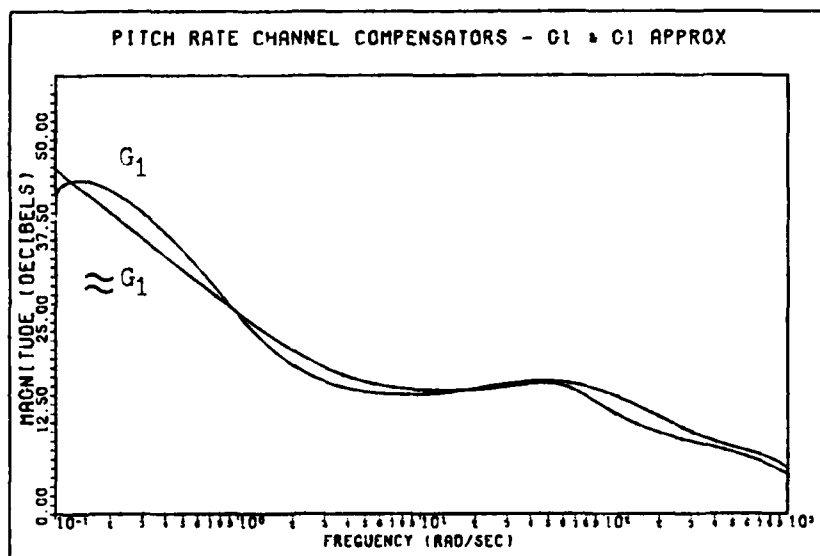


Fig. 5-1. Bode Plot Comparison of  $G_1$  and the Approximation for  $G_1$

The  $q$  responses for the four flight conditions and six CSC modes are seen in Figures 5-2a to 5-2d. Each figure represents the responses for one flight condition and the six CSC modes. The responses are examined for

are explained for compliance with the performance tolerances detailed in Chapter IV. Surface deflections are examined for saturation, and a more efficient division of control effort is investigated to reduce or eliminate saturation. Control surface initial rates are found to saturate for more cases when the input is a pulse, but representative examples with ramped inputs are also presented showing rates that are not saturated. Mr. Yin-Kuei Liao of the Chung-Shan Institute of Science and Technology, Republic of China, developed the computer program used to simulate the system of Figure 2-1. It is recommended that follow-on thesis efforts in QFT concentrate on developing a computer-aided design (CAD) package adaptable to higher-order systems. A listing of the computer programs used in the simulation is presented in Appendix F. Compliance with the specifications for  $q$  are investigated first.

#### V.2.1. Pitch Rate Command Response

Before proceeding with the  $q$  responses, it is noted that the compensator  $g_1$ , Eq. (4-28), is approximated as follows:

$$g_1 = \frac{-6.6(10^5)(s+4)(s+100)(s^2+0.013s+0.006)}{s(s+0.0223)(s+25)(s+630)^2} \quad (5-1)$$

Thus, the order of  $g_1$  is reduced from tenth to fifth-order. Equation (5-1) is obtained from a straight-line approximation of the Bode plot for Eq. (4-28). The approximation

## V. Results

### V.1 Introduction

The results of the computer simulation are presented in this chapter. The digital simulation program integrates the system equations in state-variable form to yield the desired outputs. The  $q$  and  $p$  command and cross-coupling responses are examined for compliance with performance tolerances. Control surfaces are examined for proper sense of movement and saturation. The insights that QFT affords a designer are investigated, and the results of control surface "hardover" simulations are illustrated. A brief description of the simulation program follows.

### V.2 Simulation

The system represented in Figure 2-1 is simulated with the plant, prefilter, and compensator equations in state-variable form. A digital integration routine is applied to the system over a desired time interval to yield a nonlinear, real-time simulation which includes control surface position saturation. The outputs of the simulation are  $p$ ,  $q$ , and the four control surface positions (right/left elevators and right/left flaperons). Outputs are obtained for each design flight condition and control surface configuration (CSC) mode, and the  $q$  and  $p$  responses

transmissions are then shaped on the Nichol's Chart. Simple expressions for the prefilters are derived completing the designs. Lastly, stability only designs for configurations with three simultaneous surface failures are synthesized, and further illustrates the potential of quantitative feedback theory. Chapter V presents the system simulations.

$$(1+L_{2o_e}) = \frac{(s+1.56)(s+32.5+j46.9)(s+424.6+j567)}{s(s+22.9)(s+34.6)(s+429.2+j572.2)} \quad (4-39)$$

and

$$(1+L_{1o_e}) = \frac{(s+1.996)(s+1388+j1838)(s+1426+j1914)}{s(s+1408+j1877)^2} \quad (4-40)$$

The characteristic polynomial is formed by the product of Eqs. (4-39) and (4-40). Again, the system is stable as all the zeros of Eq. (4-37) lie in the left-half-plane.

These designs for stability further illustrate the power of QFT. Loss of aircraft control is the probable result if time is required to identify and switch in compensation for the two failure cases examined in this section. No identification and switching in of compensation is required for the fixed compensators designed using QFT. This buys the aircrew valuable time; for example, time to maneuver the aircraft into more favorable ejection parameters.

#### IV.7 Summary

This chapter presents a complete description of the procedure used in the compensator designs. First, the equivalent plant expressions are derived, and the equivalent single-loop plant equations ( $Q_{ij}$ ) are obtained. Specifications are derived for the command and cross-coupling responses. Based on these specifications, bounds on the loop transmissions are obtained, and the loop

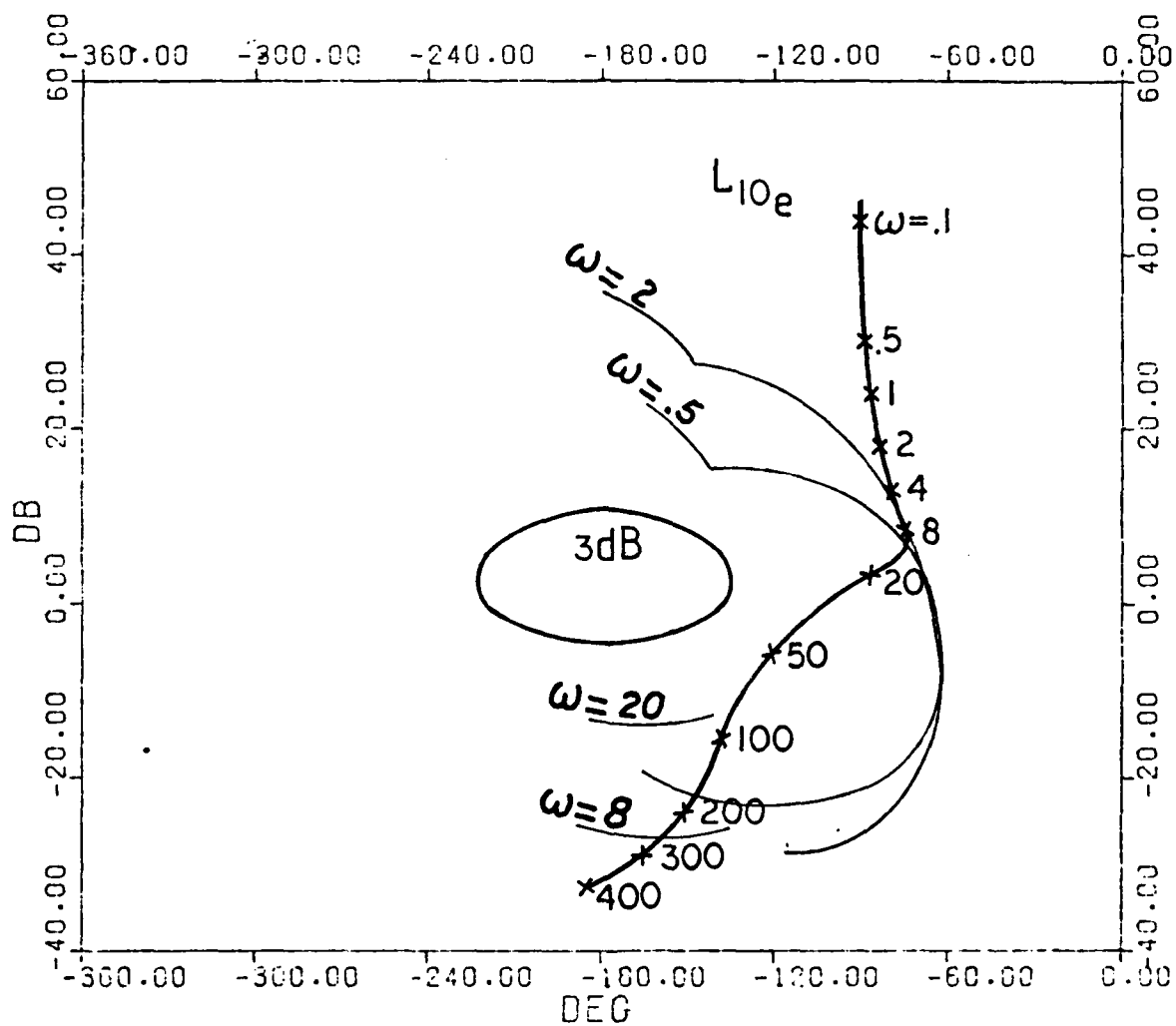


Fig. 4-13.  $L_{10e}$  and Bounds for  
Stability Only (One Elevator Only)



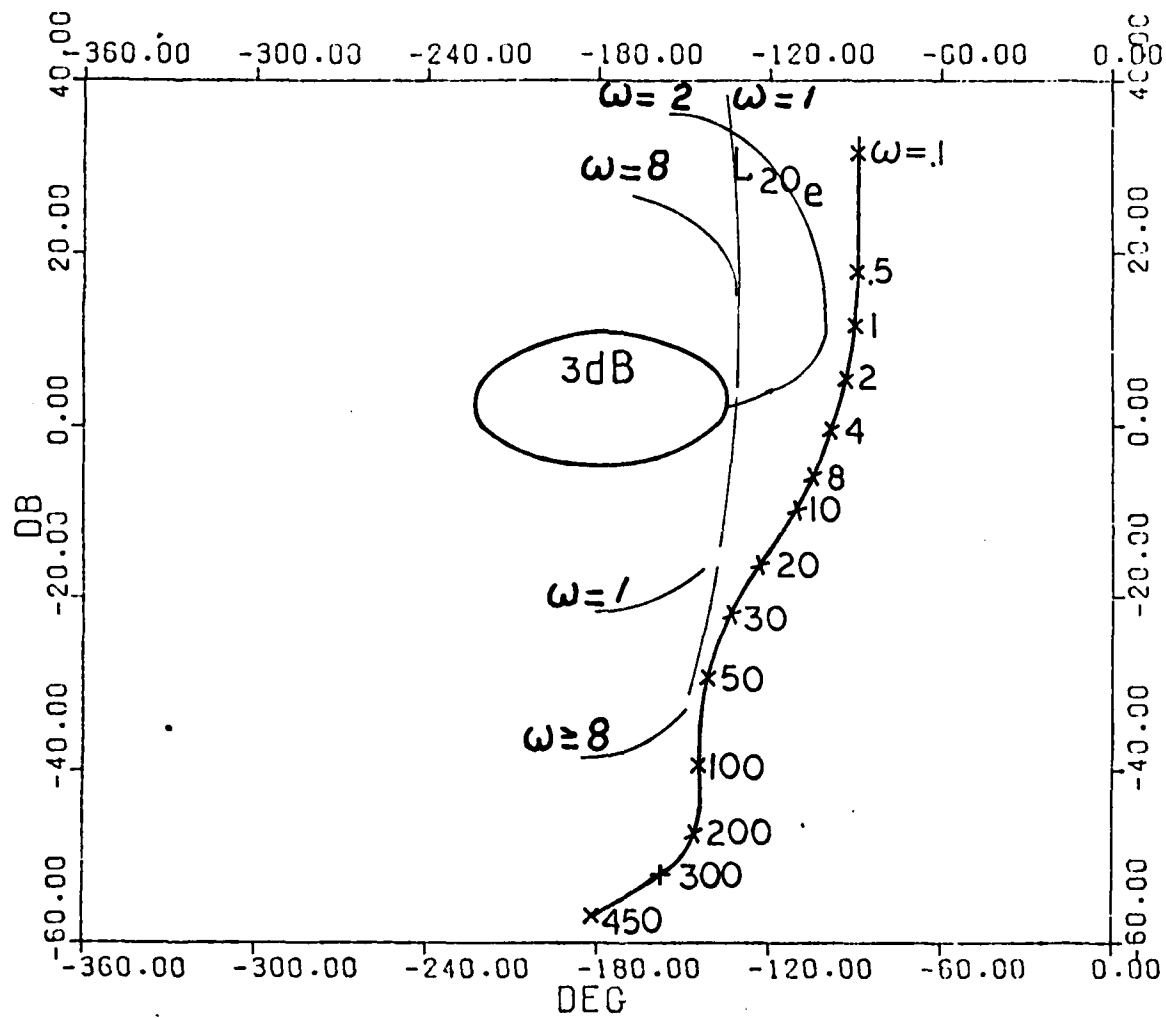


Fig. 4-12.  $L_{20e}$  and Bounds for  
Stability Only (One Elevator Only)

and

$$(1+L_{10_e}) = \frac{(s+5.7)(s+28.7 \pm j37.3)(s+278.5 \pm j371.1)}{s(s+22.7)(s+34.6)(s+281.4 \pm j375.2)} \quad (4-36)$$

The characteristic polynomial is the product of Eqs. (4-35) and (4-36), and as can be seen, all zeros are in the left-half-plane, indicating a stable system. Figures 4-12 and 4-13 show  $L_{20_e}$  and  $L_{10_e}$ , respectively, and the bounds on each.

The same procedure as above is used to design for stability when only one flaperon is controllable. In this case only  $P_{22}^a$  and  $P_{12}^a$  exist ( $P_{11}^a = P_{21}^a = P_{11}^b = P_{21}^b = P_{22}^b = P_{12}^b = 0$ ). When these substitutions are made into Eq. (3-7), the characteristic polynomial is

$$(1 + \mu P_{12}^a g_1 + P_{22}^a g_2) \quad (4-37)$$

Equation (4-37) may be factored as follows

$$(1+P_{22}^a g_2) \frac{(1+\mu P_{12}^a g_1)}{1+P_{22}^a g_2} = (1+L_{20_e})(1+L_{10_e}) \quad (4-38)$$

where,

$$L_{20_e} = P_{22}^a g_2, \text{ and } L_{10_e} = \frac{\mu P_{12}^a g_1}{1+P_{22}^a g_2}$$

The factors in Eq. (4-38) are derived in a manner similar to that used to derive the factors in Eq. (4-34). The design process yields

$$\underline{T} = \frac{\begin{bmatrix} \mu g_1 P_{12}^a & g_2 P_{12}^a \\ \mu g_1 P_{22}^a & g_2 P_{22}^a \end{bmatrix}}{1 + \mu P_{12}^a g_1 + P_{22}^a g_2} \quad (4-32)$$

The characteristic polynomial for this configuration is

$$1 + \mu P_{12}^a g_1 + P_{22}^a g_2 \quad (4-33)$$

Equation (4-33) is factored as follows:

$$(1 + P_{22}^a g_2) \cdot \frac{(1 + \mu P_{12}^a g_1)}{1 + P_{22}^a g_2} = (1 + L_{2o_e}) (1 + L_{1o_e}) \quad (4-34)$$

Where,

$$L_{2o_e} = P_{22}^a g_2, \text{ and } L_{1o_e} = \frac{\mu P_{12}^a g_1}{1 + P_{22}^a g_2}$$

Prefilters  $f_1$  and  $f_2$  in Eq. (3-7) are made equal to unity for the convenience of this analysis.

Loop transmissions are shaped separately on the Nichol's Chart for each factor in Eq. 4-34. The  $L_{2o_e}$  loop is shaped first so that the factor  $(1 + L_{2o_e})$  can be used in forming the bounds for the  $L_{1o_e}$  loop. From Total, the factors of Eq. (4-34) are

$$(1 + L_{2o_e}) = \frac{(s+4.9)(s+18.4)(s+299.7+j399.7)}{s(s+22.9)(s+300+j400)} \quad (4-35)$$

of  $L_{10}$  and  $L_{20}$ , the stability of the system is determined. Configurations for which only one elevator, and only one flaperon are controllable are examined separately.

The first configuration analyzed is the system for which only one elevator surface remains controllable. The remaining elevator and the two flaperons are failed to their neutral reference positions. The design begins by determining  $\underline{P}_e$ . Only  $P_{11}^a$  and  $P_{22}^a$  exist for this configuration ( $P_{11}^b = P_{21}^b = P_{22}^a = P_{12}^a = P_{22}^b = P_{12}^b = 0$ ). These substitutions are made into Eq. (3-6) to yield

$$\underline{P}_e = \begin{bmatrix} P_{11}^a & \mu P_{11}^a \\ P_{21}^a & \mu P_{21}^a \end{bmatrix} \quad (4-31)$$

As before, in order to determine the equivalent single-loop plant equations  $Q_{ij}$ ,  $\underline{P}_e^{-1}$  must be computed. From Eq. (4-31)  $|\underline{P}_e| = 0$ , and  $\underline{P}_e^{-1}$  does not exist. In this case the system effectively becomes a SISO system with only one output independently achievable. The  $P_{ije}$  ( $i=j$ ) plant equations are now used in the detailed design procedure.

When the elements of Eq. (4-31) are substituted into the system transfer function matrix Eq. (3-7), the system matrix for one controllable elevator becomes

time constant of  $f_{11}$ . Scheduling of  $f_{11}$  may need to be employed if the  $t_s$  is too small for some flight conditions. Close examination of the data in Table 5-2 also reveals that the difference between the maximum and minimum  $t_s$  is 2.14 sec. This exceeds the  $t_s$  tolerance of 1.5 sec by 0.64 sec. Quantitative feedback theory guarantees that the original performance tolerances will be satisfied despite the uncertainty (2). An exact reason why this has occurred is not known, but one possible explanation is that the approximation for  $g_1$  may not be adequate to satisfy the tolerances in all cases. Despite this fact,  $q$  is found to be controllable for each flight condition and CSC mode. A roll angle tolerance when  $q$  is commanded must also be satisfied.

As stated earlier in Section IV.3, it is desired that the roll angle be less than five degrees when  $q$  is commanded. The roll angle is estimated by graphically calculating the area under the  $p$  response curves resulting from cross-coupling. The cross-coupling responses are seen in Figures 5-3a to 5-3d. Only four responses are seen per figure as cross-coupling does not occur for CSC modes one and six due to the symmetry of the control surfaces for these two modes. The estimated worse case total roll angle at each flight condition is seen in Table 5-3. Inspection of this data reveals that the roll angle specification is satisfied for all cases. Control surface deflections when  $q$  is commanded are examined next.

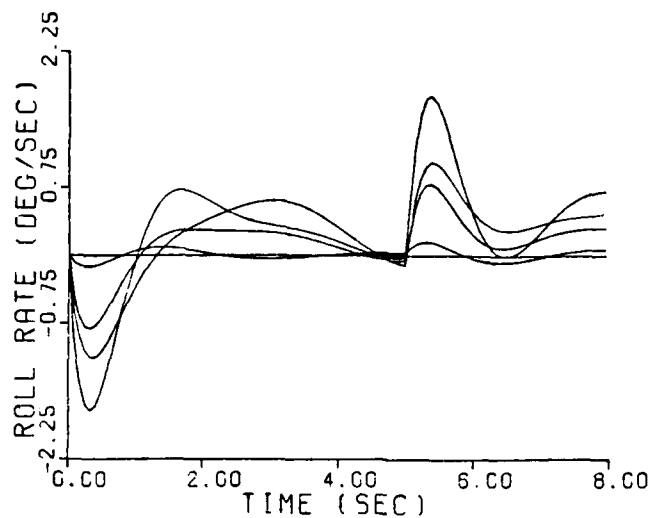


Fig. 5-3(a). Roll Rate Cross-Coupling Responses, Flight Condition One

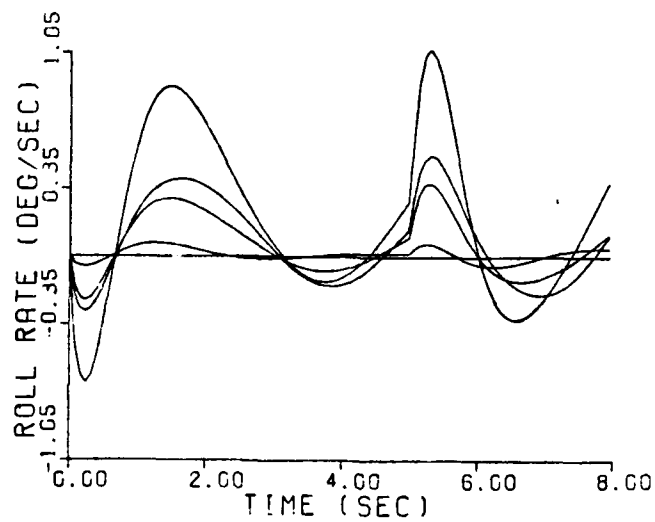


Fig. 5-3(b). Roll Rate Cross-Coupling Responses, Flight Condition Two

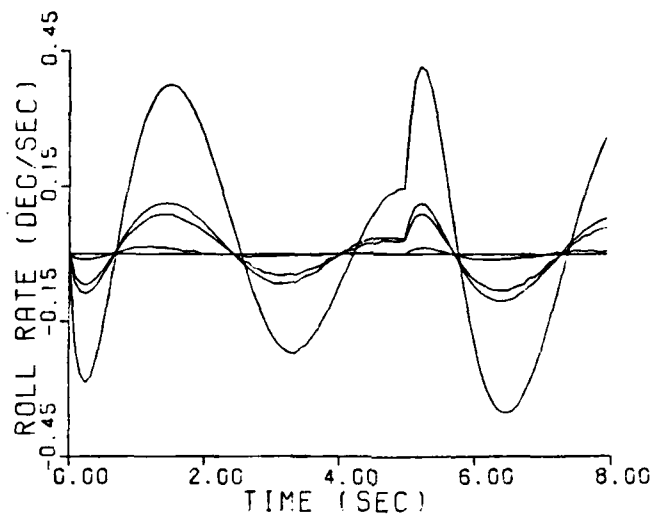


Fig. 5-3(c). Roll Rate Cross-Coupling Responses, Flight Condition Three

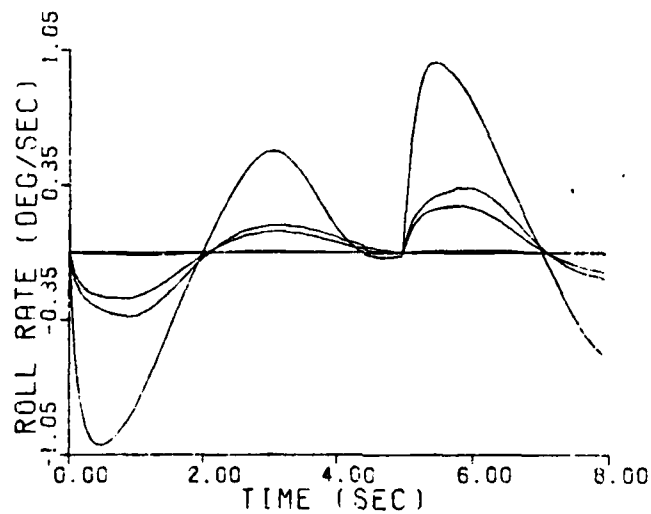


Fig. 5-4(d). Roll Rate Cross-Coupling Responses, Flight Condition Four

TABLE 5-3  
ESTIMATED ROLL ANGLES WITH  
PITCH RATE COMMANDED

Flight Condition	CSC Mode	Roll Angle (deg)
1	4	2.16
2	4	1.85
3	4	0.74
4	4	2.74

Elevator and flaperon deflections are seen in Figures 5-4a to 5-4f. Due to the large number of figures for the surface deflections, only those figures for flight condition one are presented in the main body of this thesis. The remaining figures are displayed in Appendix G. Negative deflection angles indicate that lift is being generated in the negative direction (down), and vice versa for positive deflection angles. The sign convention used is detailed in Reference 6. Figure 5-4a (CSC mode 1) shows that surfaces are moving in the proper directions to generate positive  $q$ . The negative deflections generate negative lift at the elevator and flaperon stations causing the nose of the aircraft to rotate up (positive  $q$ ). Although not as obvious in some of the figures, surface movements appear to be in the proper sense to generate positive  $q$ . Control surface position and rate limit data



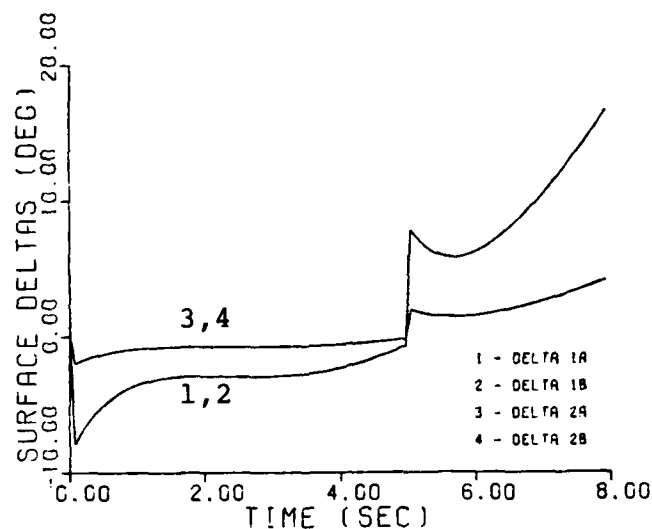


Fig. 5-4(a). Elevator and Flaperon Deflections  
for Pitch Rate, CSC Mode One and  
Flight Condition One

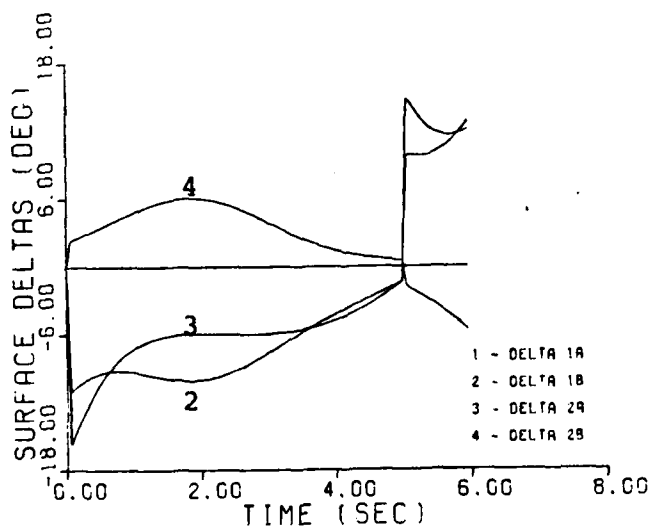


Fig. 5-4(b). Elevator and Flaperon Deflections  
for Pitch Rate, CSC Mode Two and  
Flight Condition One

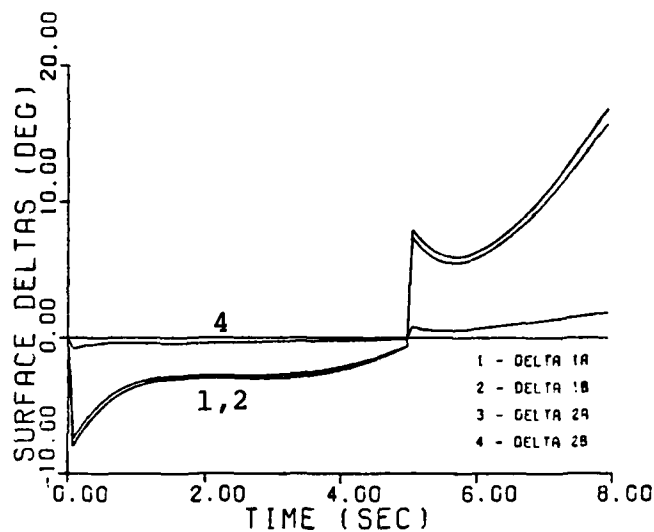


Fig. 5-4(c). Elevator and Flaperon Deflections  
for Pitch Rate, CSC Mode Three and  
Flight Condition One

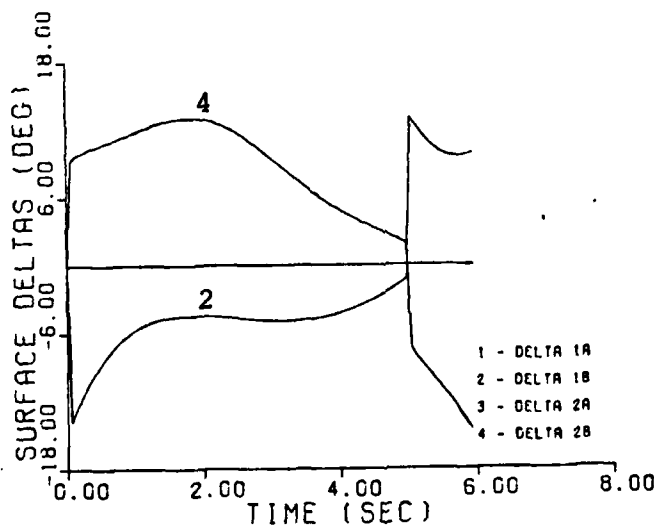


Fig. 5-4(d). Elevator and Flaperon Deflections  
for Pitch Rate, CSC Mode Four and  
Flight Condition One

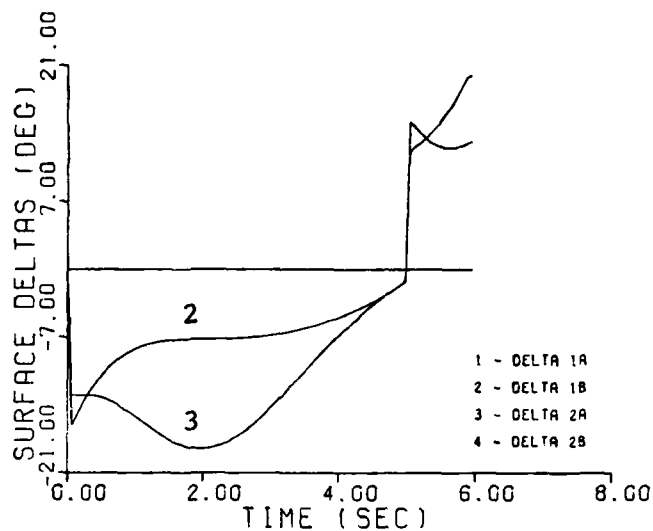


Fig. 5-4(e). Elevator and Flaperon Deflections  
for Pitch Rate, CSC Mode Five and  
Flight Condition One

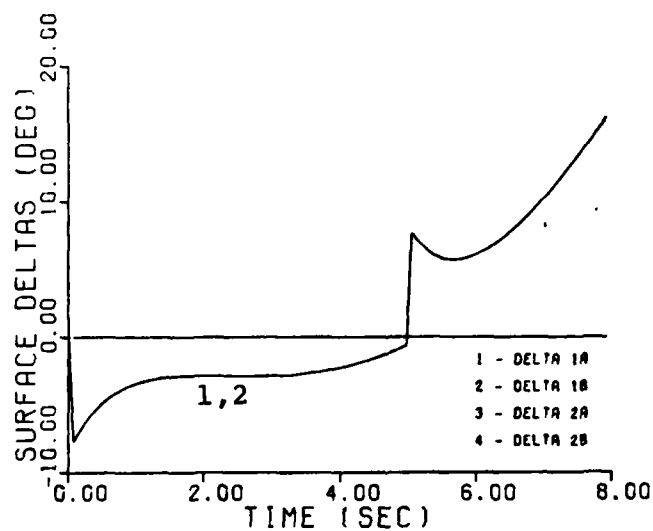


Fig. 5-4(f). Elevator and Flaperon Deflections  
for Pitch Rate, CSC Mode Six and  
and Flight Condition One

are taken from Reference 6, and repeated in Table 5-4. Inspection of Figures 5-4a to 5-4f show that surface position saturation does not occur for the q command at flight condition one. Initial surface rates are seen to be quite high, and are estimated to saturate or nearly saturate for the pulse input. Saturation is further addressed in a later section of this chapter. In summary, control surface movements appear to be in the proper sense to generate positive q for all flight conditions and CSC modes.

TABLE 5-4  
SURFACE POSITION AND RATE LIMITS

Surface	Position Limit (deg)	Rate Limit (deg/sec)
Elevators	±25	60
Flaperons	±20	52

#### V.2.2 Roll Rate Command Response

The p responses for the four flight conditions and CSC modes are seen in Figures 5-5a to 5-5d. Each response is examined for compliance with the tolerances listed in Table 4-2, repeated here as Table 5-5.

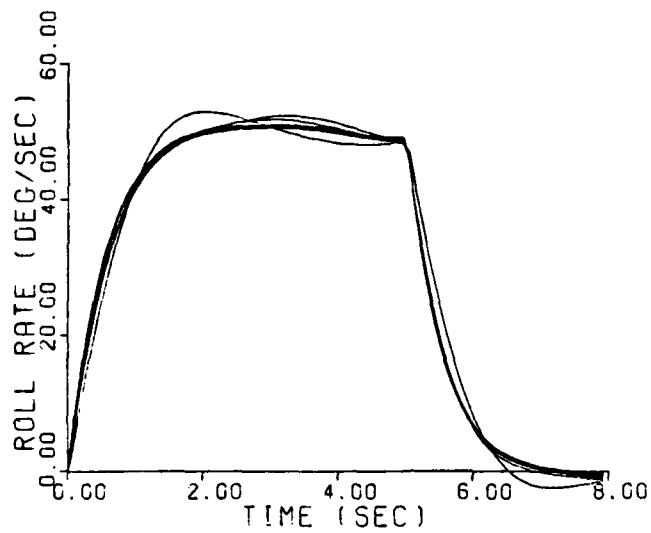


Fig. 5-5(a). Roll Rate Responses,  
Flight Condition One

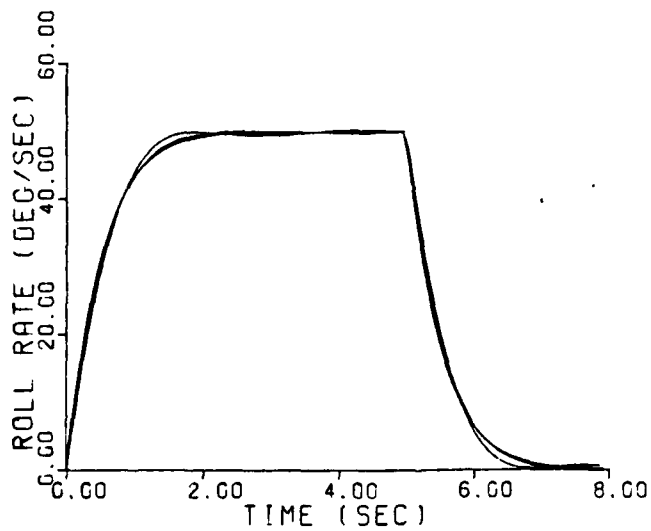


Fig. 5-5(b). Roll Rate Responses,  
Flight Condition Two

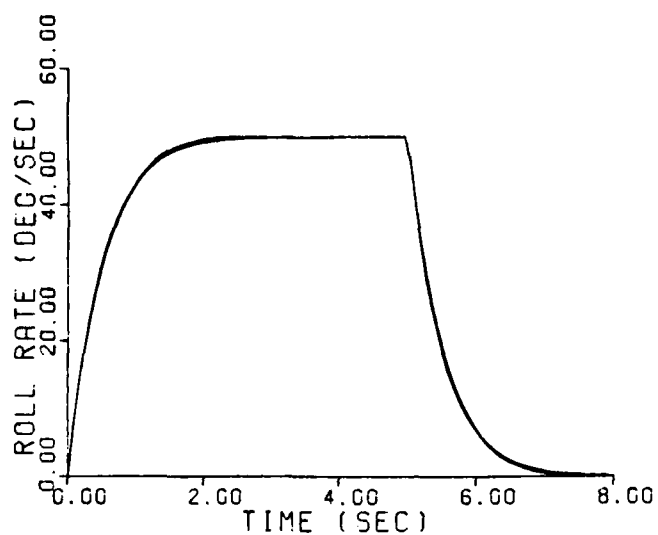


Fig. 5-5(c). Roll Rate Responses,  
Flight Condition Three

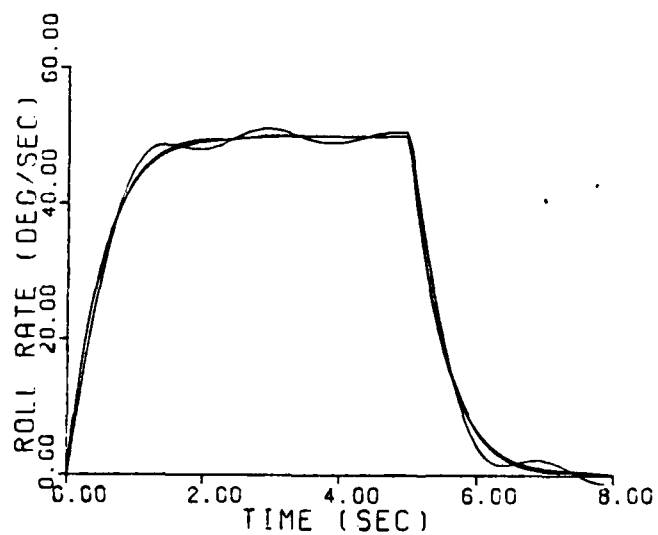


Fig. 5-5(d). Roll Rate Responses,  
Flight Condition Four

TABLE 5-5  
FIGURE OF MERIT TOLERANCES  
FOR ROLL RATE

Settling Time $t_s$ (sec)	Rise Time $t_r$ (sec)	Peak Value $M_p$ (deg/sec)
3.0 - 6.0	0.65 - 3.23	$\leq 55$

The figures of merit calculated for each p response are listed in Table 5-6. As is necessary for  $g_1$ , the compensator  $g_2$ , Eq. (4-16), is approximated in the same manner as  $g_1$ . The approximation for  $g_2$  is

$$g_2 = \frac{-3.68(10^4)(s+3)(s+900)(s^2+2s+4)}{s(s+0.25)(s+50)(s+158)(s+3750)} \quad (5-2)$$

The original ninth-order equation for  $g_2$  is reduced to an effective fifth-order equation. Figure 5-6 shows a Bode plot comparison of Eqs. (4-16) and (5-2). The data in Table 5-6 is examined for compliance with performance tolerances.

The data in Table 5-6 indicates that performance tolerances for  $t_s$ ,  $t_r$ , and  $M_p$  are satisfied for all but, possibly, one case. The  $t_s$  for CSC mode 6 and flight condition one is greater than five seconds due to the oscillatory nature of the response. Also, most of the responses have settling times that are faster than desired. These responses may be slowed by using a larger time constant

TABLE 5-6

## ROLL RATE RESPONSE FIGURES OF MERIT

Flight Condition	CSC Mode	Settling Time $t_s$ (sec)	Rise Time $t_r$ (sec)	Peak Value $M_p$ (deg/sec)
1	1	4.25	1.15	50.7
	2	4.16	1.22	51.1
	3	3.84	1.17	51.0
	4	3.84	1.30	51.9
	5	4.00	1.42	52.4
	6	5.00+	1.14	53.0
2	1	1.85	1.06	50.1
	2	4.16	1.21	51.1
	3	1.80	1.10	50.0
	4	1.76	1.10	50.0
	5	1.70	1.02	49.9
	6	1.50	1.00	50.1
3	1	1.92	1.10	50.0
	2	1.88	1.09	50.0
	3	1.86	1.10	50.0
	4	1.76	1.16	50.0
	5	1.84	1.10	50.0
	6	1.76	1.03	50.0
4	1	1.92	1.00	50.0
	2	1.98	1.04	50.1
	3	1.88	1.07	50.0
	4	4.48	0.95	51.1
	5	1.94	1.05	50.1
	6	1.76	1.06	50.1



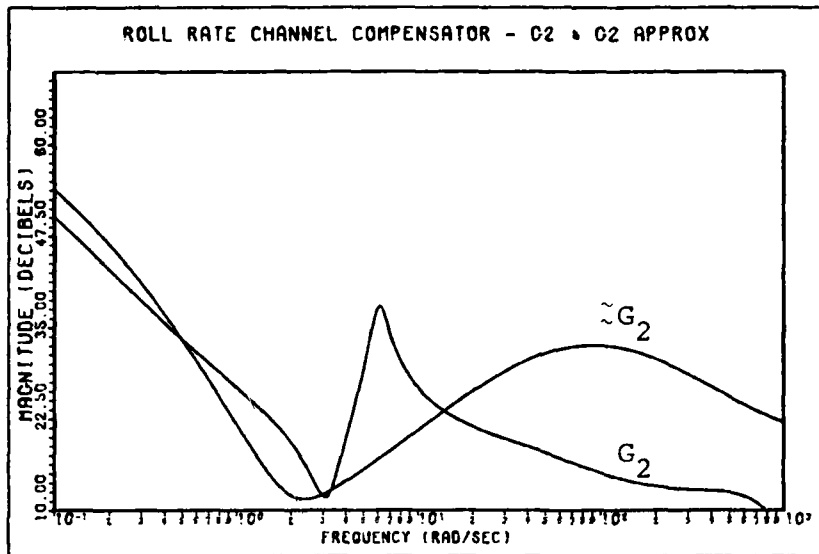


Fig. 5-6. Bode Plot Comparison of  $G_2$  and the Approximation for  $G_2$

in the prefilter  $f_{22}$ . A higher-order prefilter can be used to shape the responses to obtain more desirable figures or merit. This is not attempted in this study due to a restriction on the number of equations that the simulation program can accommodate. The  $q$  resulting from the  $p$  command is examined for compliance with performance specifications.

The pitch angle due to cross-coupling is estimated in the same way that roll angle due to cross-coupling is estimated. It is desired that the pitch angle remain less than three degrees during the  $p$  command. The  $q$

cross-coupling responses are shown in Figures 5-7a to 5-7d, and Table 5-7 lists the worse case pitch angle estimated at each flight condition.

TABLE 5-7  
ESTIMATED PITCH ANGLE WITH  
ROLL RATE COMMANDED

Flight Condition	CSC Mode	Pitch Angle (deg)
1	5	-3.10
2	4	-0.70
3	4	-1.00
4	4	-1.25

The magnitude of the pitch angle for CSC mode 5 and flight condition 1 is seen to slightly exceed 3 degrees. If the accuracy of the pitch angle estimations is conservatively estimated at 5 percent, the performance specification for pitch angle is satisfied. Control surface deflections due to the p command are estimated for proper sense of movement and saturation.

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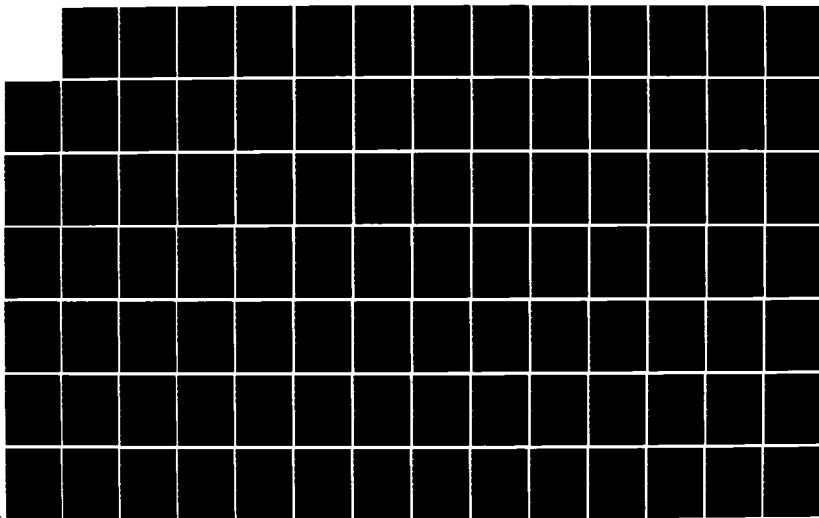
FLIGHT CONTROL SYSTEM RECONFIGURATION DESIGN USING  
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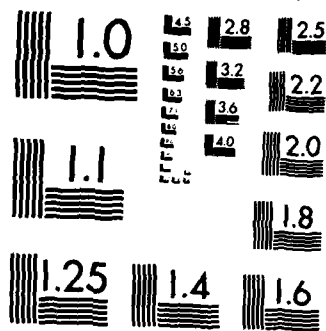
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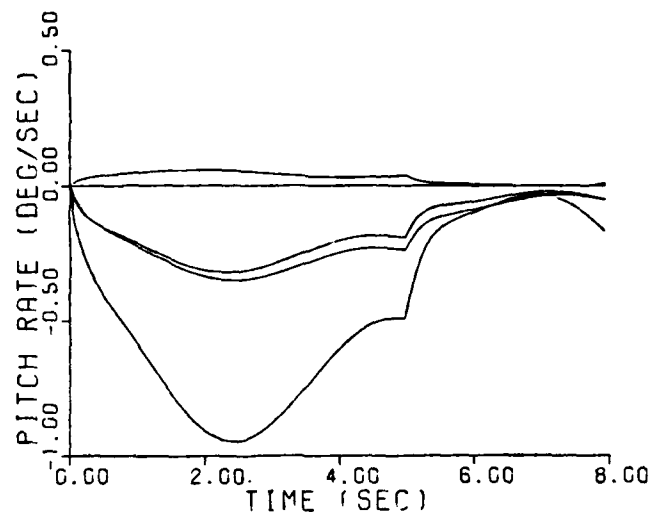


Fig. 5-7(a). Pitch Rate Cross-Coupling Responses, Flight Condition One

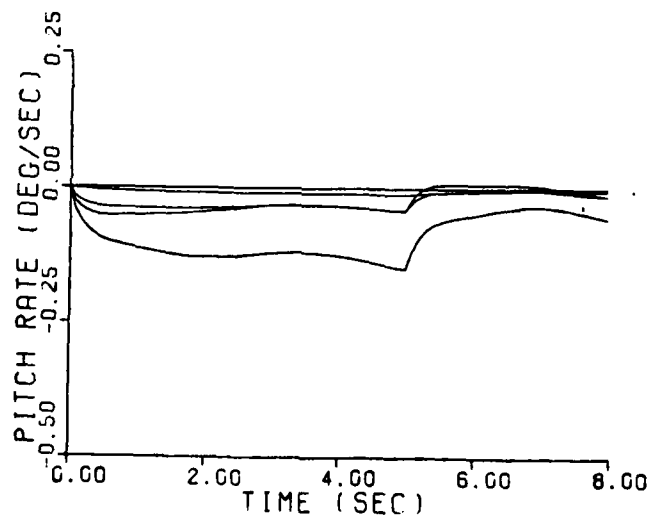


Fig. 5-7(b). Pitch Rate Cross-Coupling Responses, Flight Condition Two

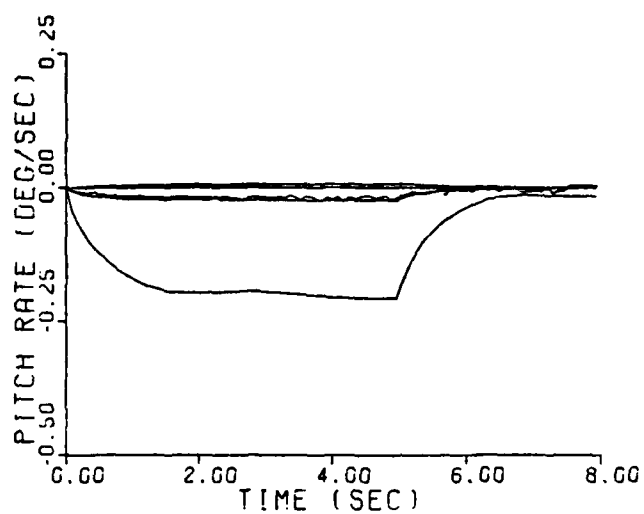


Fig. 5-7(c). Pitch Rate Cross-Coupling Responses, Flight Condition Three

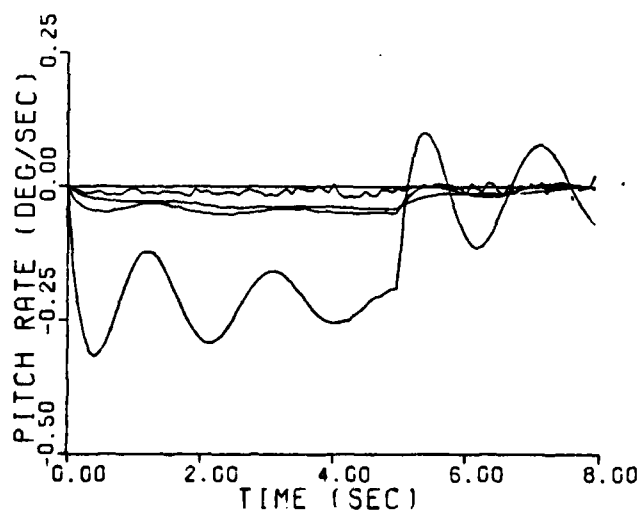


Fig. 5-7(d). Pitch Rate Cross-Coupling Responses, Flight Condition Four

Elevator and flaperon deflections for the p command at flight condition one are seen in Figures 5-8a to 5-8f. As is seen in Figure 5-8a, the surface pairs are moving differentially in the proper sense to generate positive roll rate. The flaperons are also commanded in excess of their 20 degree limit, while the elevators are well below their limit of 25 degrees. This is essentially the case for the remaining figures in this sequence. It is obvious from Figures 5-8 that more of the control effort needs to be shared by the elevators to reduce flaperon movement. This point is addressed later when saturation is discussed. Initial surface rates for all surfaces are near or at saturation for the pulse input; these rates are greatly reduced when the input is ramped, as is seen later. The surface deflection figures for p at flight conditions two, three, and four are displayed in Appendix G.

It is concluded from the pitch rate and roll rate results that the compensators  $g_1$  and  $g_2$  provide robust control for all flight conditions and CSC modes for which the system is designed despite minor qualifications for the pitch rate results. It is also reasonable to conclude that the system provides robust control for all points encompassed by the design flight conditions. Saturation is the next topic of discussion.

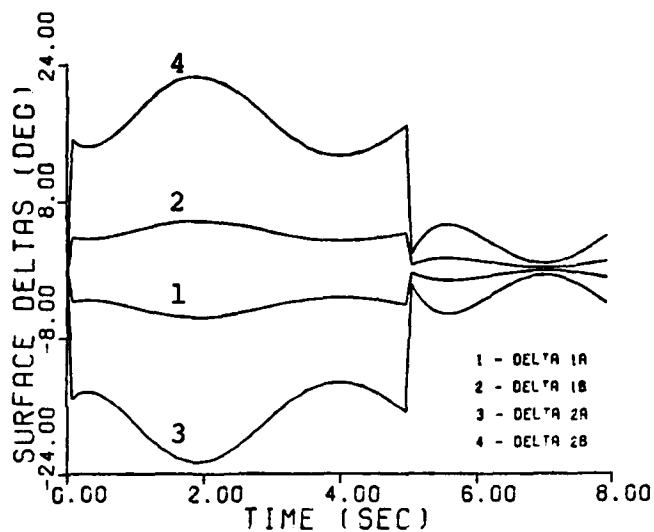


Fig. 5-8(a). Elevator and Flaperon Deflections  
for Roll Rate, CSC Mode One and  
Flight Condition One

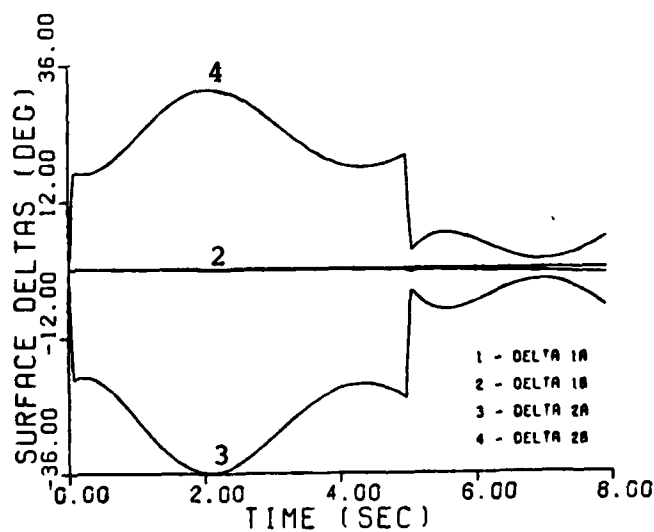


Fig. 5-8(b). Elevator and Flaperon Deflections  
for Roll Rate, CSC Mode Two and  
Flight Condition One



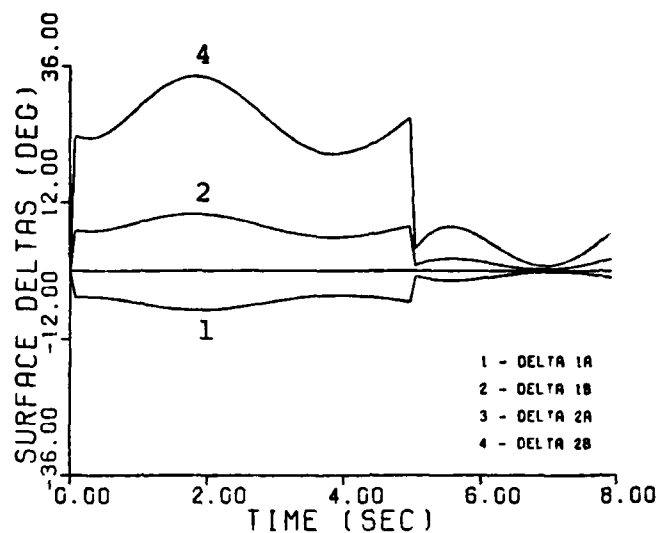


Fig. 5-8(c). Elevator and Flaperon Deflections  
for Roll Rate, CSC Mode Three and  
Flight Condition One

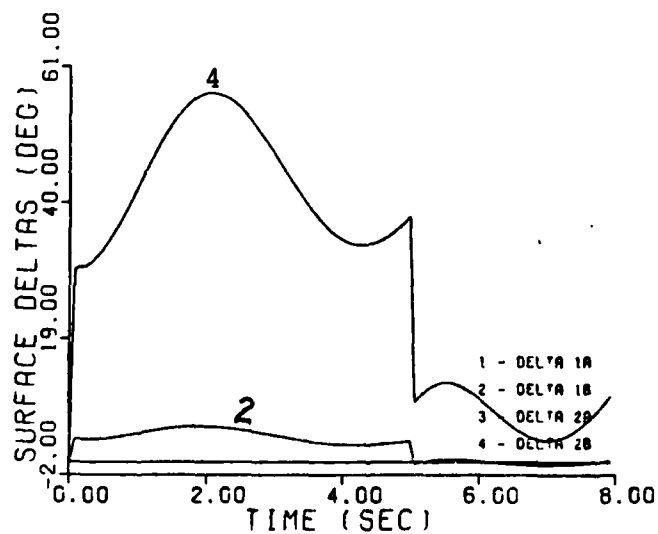


Fig. 5-8(d). Elevator and Flaperon Deflections  
for Roll Rate, CSC Mode Four and  
Flight Condition One

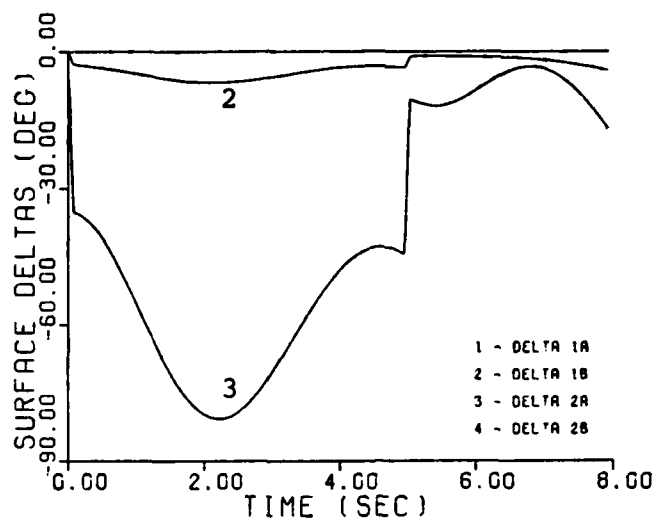


Fig. 5-8(e). Elevator and Flaperon Deflections  
for Roll Rate, CSC Mode Five and  
Flight Condition One

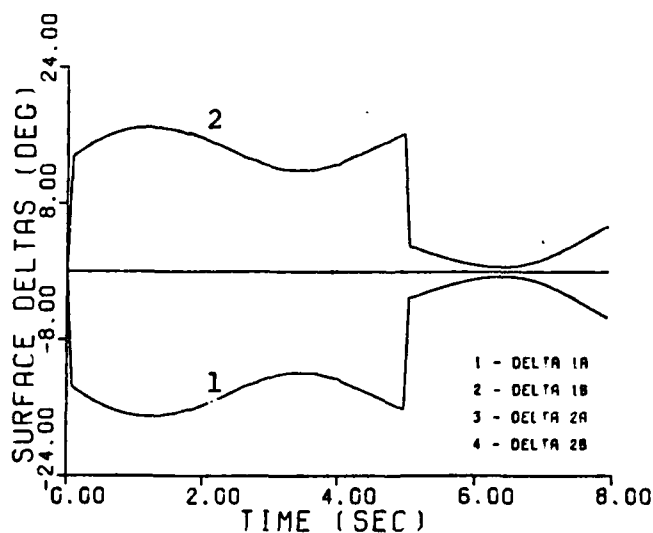


Fig. 5-8(f). Elevator and Flaperon Deflections  
for Roll Rate, CSC Mode Six and  
Flight Condition One

### V.3 Saturation

Different results are obtained for several of the cases when the provision for position saturation is included in the simulation program. Flight condition one, CSC modes one and two are taken as cases in point. The surface deflections when  $p$  is commanded for CSC mode one without saturation are presented in Figure 5-8a, and it is evident that the flaperons show deflections in excess of their limit. The same simulation is run with position saturation included, and the results are seen in Figure 5-9. The flaperons are seen to saturate, and the elevator deflections increase to generate the commanded roll rate. The  $p$  responses with and without saturation are seen in Figure 5-10. Note the more oscillatory nature of the response with saturation. The result is much more dramatic for CSC mode two; Figure 5-11 displays the responses for this CSC mode with and without saturation. Cross-coupling is found to increase significantly when surfaces saturate, as is seen in Figure 5-12a. Surface deflections corresponding to Figure 5-12a are seen in Figure 5-12b. It is obvious from these illustrations that position saturation reduces the stability of the system. Saturation can be reduced or eliminated by better distribution of the control effort.

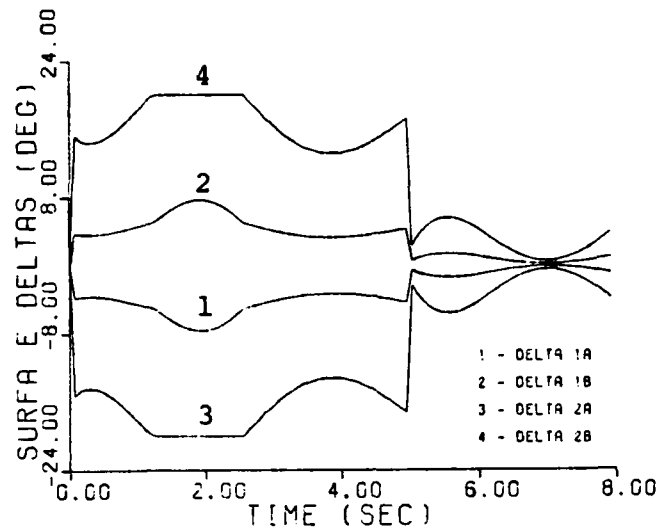


Fig. 5-9. Elevator and Flaperon Deflections  
With Saturation, CSC Mode One  
and Flight Condition One

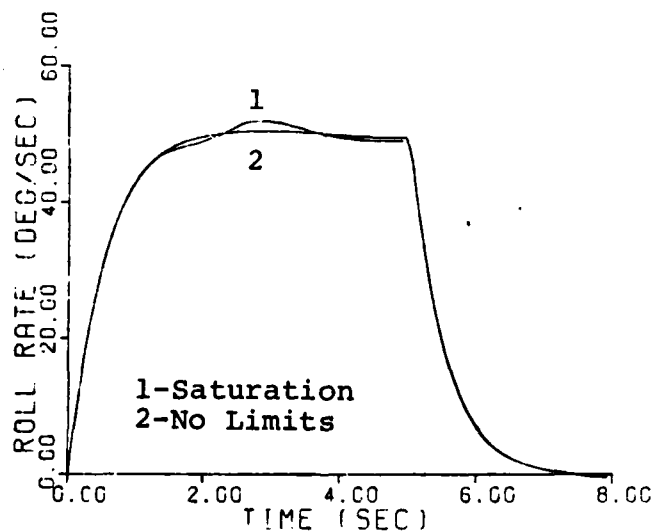


Fig. 5-10. Roll Rate Response With and  
Without Saturation, CSC Mode One  
and Flight Condition One

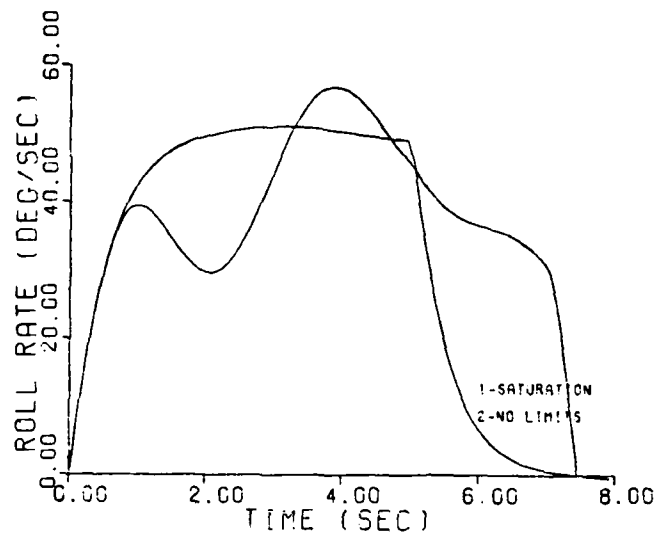


Fig. 5-11. Roll Rate Response With and Without Saturation, CSC Mode Two and Flight Condition One

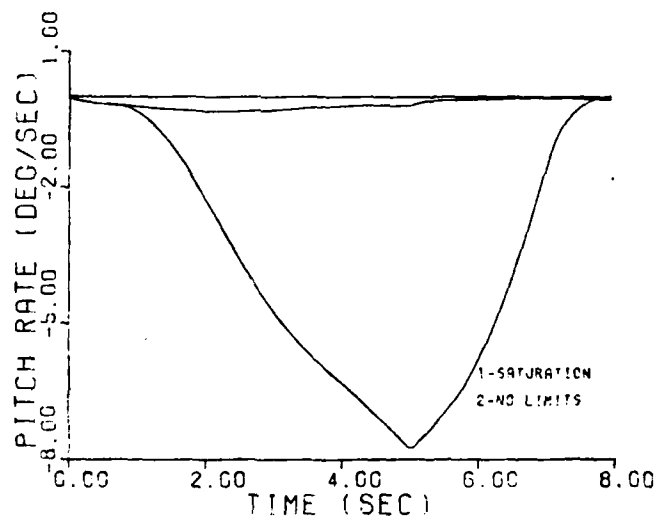


Fig. 5-12(a). Pitch Rate Cross-coupling Response With and Without Saturation, CSC Mode Two and Flight Condition One

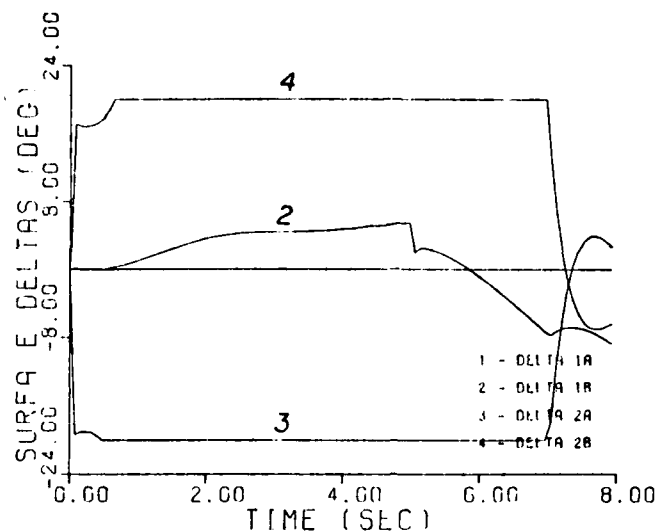


Fig. 5-12(b). Elevator and Flaperon Deflections for CSC Mode Two and Flight Condition One

The division of control effort between the elevators and flaperons may be varied by changing the constants  $\mu_{12}$  and  $\mu_{21}$  in Figure 2-1. As discussed in Section III.2, these constants are arbitrarily set at 0.25. Increasing  $\mu_{12}$  causes the elevators to share more of the effort in generating  $p$ . This is seen in Figure 5-13 where  $\mu_{12}$  is increased to 0.5. By comparing Figure 5-13 with Figure 5-9, it is seen that the flaperons no longer saturate, as more of the command is being generated by the elevators. For a reconfiguration strategy to be effective, some sort of scheme to optimize the division of control surface effort is essential. It is recommended that

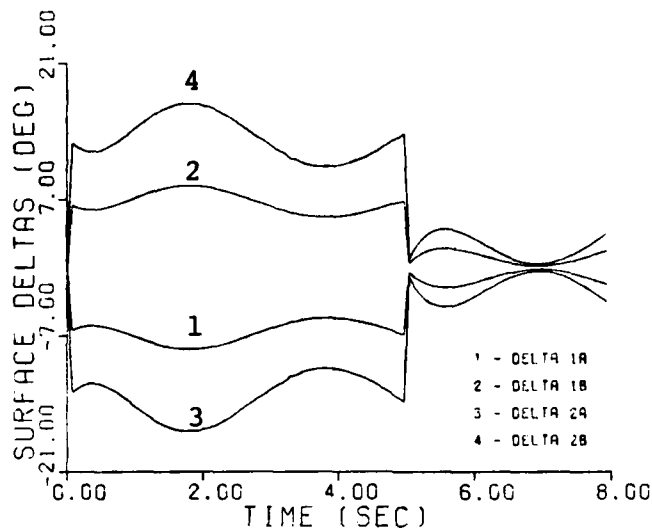


Fig. 5-13. Elevator and Flaperon Deflections  
for Roll Rate with  $\mu_{12} = 0.5$ , CSC Mode One  
and Flight Condition One

follow-on thesis efforts in flight control system reconfiguration investigate strategies to optimize the division of control effort. Appendix B contains theory developed by Dr. Horowitz to optimize the division of control effort. Initial surface rates are noted earlier to be high.

So far, all inputs have been pulse commands. Figures 5-14 and 5-15 illustrate the effect on surface rates when the inputs are ramped. In Figure 5-14 the input is a clipped ramp with a slope of  $20 \text{ deg/sec}^2$  (0.5 sec to peak value). The elevator and flaperon initial rates are found to be well below saturation levels. The same is

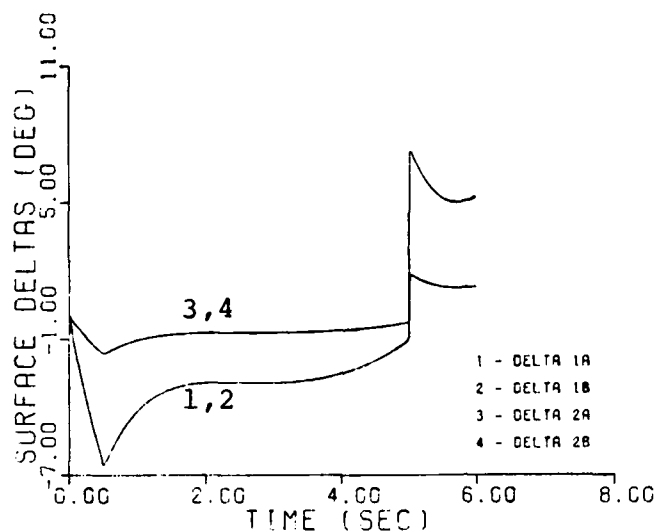


Fig. 5-14. Elevator and Flaperon Deflections for Pitch Rate with a Ramped Input, 0.5 sec to Peak Value (CSC Mode One and Flight Condition One)

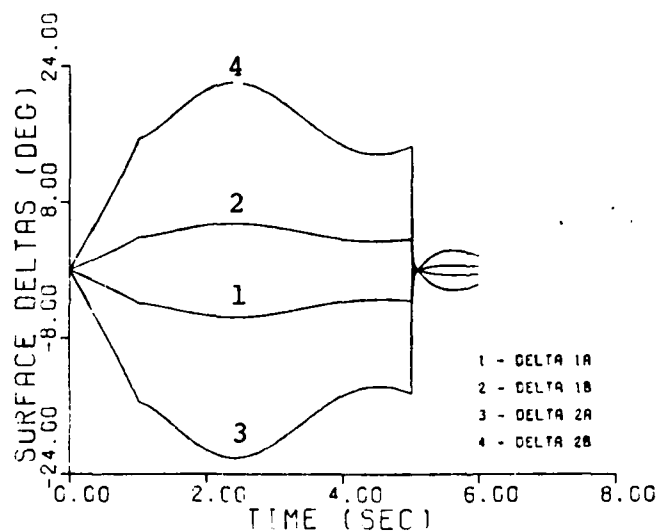


Fig. 5-15. Elevator and Flaperon Deflections for Roll Rate with a Ramped Input, 1 sec to Peak Value (CSC Mode One and Flight Condition One)



TABLE A-2

AIRCRAFT DATA FOR FLIGHT CONDITION TWO  
(Mach = 0.6 and 30,000 feet)

$$\bar{q} \text{ (dynamic pressure--lbs/ft}^2\text{)} = 158.91$$

$$V_T \text{ (trim velocity--ft/sec)} = 596.91$$

$$\alpha_T \text{ (trim angle-of-attack--deg)} = 4.705$$

---

Longitudinal Body Axis Primed Dimensional Derivatives

---

$Z_\alpha' = -0.526422$	$M_\alpha' = 2.52708$	$X_\alpha' = 23.0402$
$Z_{\delta_e}' = -0.066156$	$M_{\delta_e}' = -5.86214$	$X_{\delta_e}' = 3.17035$
$Z_{\delta_f}' = -0.111711$	$M_{\delta_f}' = -0.211773$	$X_{\delta_f}' = -2.09855$
$Z_q' = 0.997184$	$M_q' = -0.341902$	$X_q' = -48.8785$
$Z_u' = -0.000109$	$M_u' = -0.000337$	$X_u' = -0.005142$
$Z_\theta' = -0.004425$	$M_\theta' = 0.000313$	$X_\theta' = -32.0915$

---

Lateral-Directional Body Axis Primed Dimensional Derivatives

---

$N_\beta' = 2.29583$	$L_\beta' = -19.2246$	$Y_\beta' = -0.154099$
$N_p' = -0.000888$	$L_p' = -0.893601$	$Y_p' = 0.082387$
$N_r' = -0.278676$	$L_r' = 0.318845$	$Y_r' = -0.998322$
$N_{\delta_r}' = -1.96651$	$L_{\delta_r}' = 3.92325$	$Y_{\delta_r}' = 0.021165$
$N_{\delta_a}' = -0.268403$	$L_{\delta_a}' = -17.4468$	$Y_{\delta_a}' = 0.000357$
$N_{\delta_{DT}}' = -1.50547$	$L_{\delta_{DT}}' = -13.5832$	$Y_{\delta_{DT}}' = 0.014398$
$N_{\delta_c}' = 1.51008$	$L_{\delta_c}' = 0.414519$	$Y_{\delta_c}' = 0.007335$

---

TABLE A-1  
AIRCRAFT DATA FOR FLIGHT CONDITION ONE  
(Mach = 0.2 and 30 feet)

$\bar{q}$ (dynamic pressure--lbs/ft <sup>2</sup> ) = 59.18		
$V_T$ (trim velocity--ft/sec) = 223.26		
$\alpha_T$ (trim angle-of-attack--deg) = 14.98		
Longitudinal Body Axis Primed Dimensional Derivatives		
$Z_{\alpha}' = -0.459802$	$M_{\alpha}' = 0.542375$	$X_{\alpha}' = 12.5375$
$Z_{\delta_e}' = -0.074045$	$M_{\delta_e}' = -2.23634$	$X_{\delta_e}' = 0.357830$
$Z_{\delta_f}' = -0.061526$	$M_{\delta_f}' = 0.241866$	$X_{\delta_f}' = -1.97404$
$Z_q' = 0.995737$	$M_q' = -0.633651$	$X_q' = -58.0974$
$Z_u' = -0.000786$	$M_u' = -0.000651$	$X_u' = 0.002886$
$Z_{\theta}' = -0.037268$	$M_{\theta}' = -0.000058$	$X_{\theta}' = -31.1064$
Lateral-Directional Body Axis Primed Dimensional Derivatives		
$N_{\beta}' = 0.919806$	$L_{\beta}' = -11.0532$	$Y_{\beta}' = -0.143895$
$N_p' = -0.003864$	$L_p' = -0.934188$	$Y_p' = 0.261703$
$N_r' = -0.257624$	$L_r' = 1.03044$	$Y_r' = -0.994485$
$N_{\delta_r}' = -0.814818$	$L_{\delta_r}' = 1.42280$	$Y_{\delta_r}' = 0.021662$
$N_{\delta_a}' = 0.075009$	$L_{\delta_a}' = -4.47725$	$Y_{\delta_a}' = 0.010422$
$N_{\delta_{DT}}' = -0.603086$	$L_{\delta_{DT}}' = -4.28449$	$Y_{\delta_{DT}}' = 0.022018$
$N_{\delta_c}' = 0.328110$	$L_{\delta_c}' = -0.296006$	$Y_{\delta_c}' = 0.006234$

Appendix A: Flight Parameters and  
Aerodynamic Derivatives

Flight parameters and aerodynamic derivatives are presented in this appendix for the four design flight conditions. These data are taken directly from Reference 6, and repeated here for the convenience of the reader. An extensive aerodynamic package was developed by the General Dynamics Corporation for the AFTI/F-16. Derivatives are formulated in the body axis system, and have the units of radians, radians/sec, or ft/sec; i.e.,  $X_{\theta}'$  (radians),  $X_q'$  (radians/sec),  $X_u'$  (ft/sec), etc.

### VI.3 Recommendations

The following recommendations follow directly from the preceding conclusions:

1. Research using quantitative feedback theory for the design of multivariable flight control laws should continue, particularly, for reconfigurable flight control systems. Future investigations should be for systems of third or higher-order. Optimizing the division of control effort and digital control are fundamental prerequisites for a practical reconfigurable flight control system, and should be included in future research efforts. Also, a more accurate aircraft model should be developed to include coupling in the plant matrix.

2. A computer-aided design package should be developed for quantitative feedback theory. This package should include, but not be limited to the following capabilities: Bode plot, Nichol's Chart, plant template formulation, loop transmission boundary derivation, loop transmission shaping, curve fitting, and system simulation. In addition to allowing more difficult design problems to be solved, these capabilities would greatly speed up the design process and result in more precise control laws.

systems may be significant. A fixed-compensator design has the potential to significantly minimize the complexity of fault identification schemes that may be required for a reconfigurable flight control system. The next section presents some of the conclusions that may be drawn from this thesis.

## VI.2 Conclusions

1. Procedures developed from quantitative feedback theory are effective in the design of multivariable control laws for normal or impaired flight control systems. The technique reveals important insights to the designer about compensation requirements. This fixed-compensator design approach may significantly reduce the anticipated complexity of reconfigurable flight control systems.

2. The application of quantitative feedback theory in the design of multivariable control laws is a reasonably straightforward process. The technique is relatively simple once the basic theory is understood, and the designer has some practice in its use.

3. The controllers developed in this thesis provide robust control for all flight control surface configurations and flight conditions for which the design is synthesized.

4. An interactive computer-aided design package should be developed for quantitative feedback theory so that design problems of larger scope can be attempted.

## VI. Conclusions and Recommendations

### VI.1 Discussion

The primary purpose of this thesis is to demonstrate the effectiveness of quantitative feedback theory as applied to flight control system reconfiguration. Compensators are synthesized to be robust for an unimpaired flight control system as well as for several different cases in which the flight control system is severely impaired. The compensators are also to be robust for a large portion of the aircraft flight envelope. Quantitative feedback theory is applied to the linearized, small perturbation equations of motion for a longitudinally unstable aircraft (AFTI/F-16). The application of quantitative feedback theory is found to be relatively straightforward for this difficult multiple input-multiple output problem. The transparency of the technique provides important insights into the compensator requirements from the initial stages of the design process. The results of this thesis demonstrate that quantitative feedback theory is well suited for flight control system reconfiguration design. The fixed-compensator designs are seen to provide satisfactory control at all design points. The importance of fixed-compensator designs in reconfigurable flight control

compensator. Control surface "hardover" is also investigated, and simulation results for this condition further illustrate the utility of quantitative feedback theory. Recommendations and conclusions are presented in the following chapter.

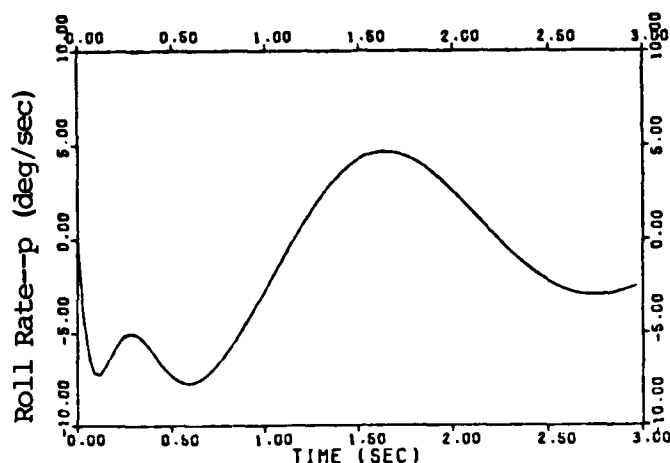


Fig. 5-19(b). Roll Rate Response from Flaperon  
Hardover, Flight Condition Two

#### V.6 Summary

This chapter presents a description of the simulation program and the results for the pitch rate and roll rate commands. From an examination of the command and cross-coupling responses it is determined that the compensators  $g_1$  and  $g_2$  provide reasonably robust control over the design flight conditions and control surface configuration modes. Saturation is examined, and the necessity to optimize the division of control effort is evident. Quantitative feedback theory is seen to provide the designer with valuable insights prior to the actual synthesis of the



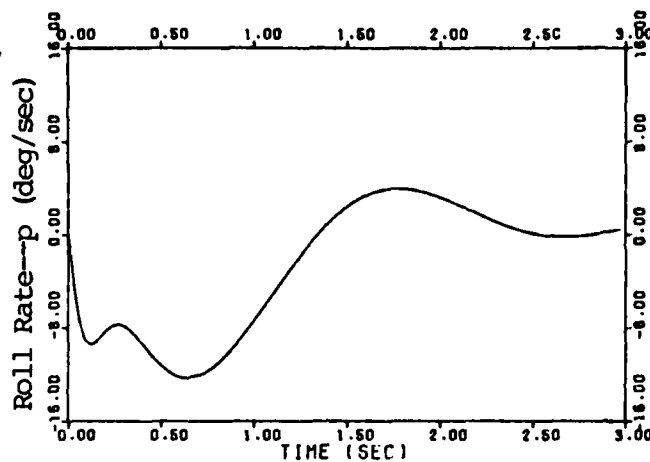


Fig. 5-18(b). Roll Rate Response from Elevator Hardover, Flight Condition Two

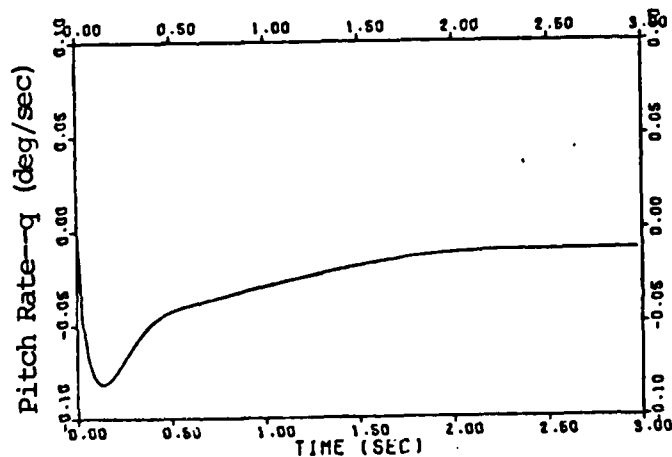


Fig. 5-19(a). Pitch Rate Response from Flaperon Hardover, Flight Condition Two

example, let  $\delta_1^a$  fail to the hardover position, the result-output vector  $(P_{11}^a \delta_{1_{\max}}^a, P_{21}^a \delta_{1_{\max}}^a)$  is treated as a disturbance to be attenuated. The equivalent plant transfer function matrix  $\underline{P}_e$  for this condition then becomes identical to  $\underline{P}_e$  for  $\delta_1^a = 0$  (CSC mode 2) since  $\delta_1^a$  is no longer controllable. Mr. Yin-Kuei Liao has simulated separate elevator and flaperon hardovers using the compensators derived in Chapter IV. Simulation results are seen in Figures 5-18 to 5-19. The system is seen to effectively attenuate the hardover surface's outputs. The results of this section further demonstrate the utility of QFT for the challenge of FCS reconfiguration. See Appendix B for Dr. Horowitz's explanation of the hardover surface problem.

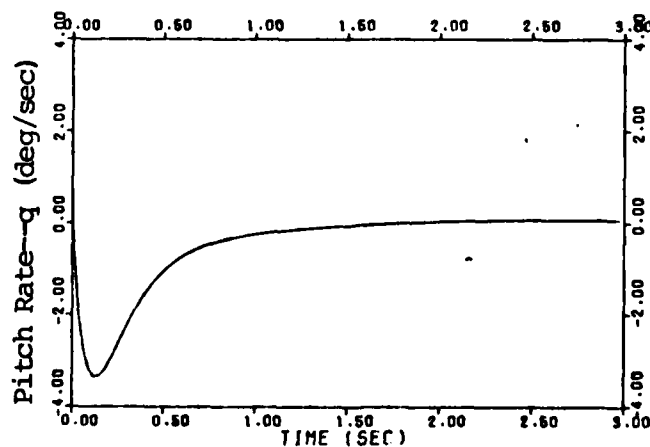


Fig. 5-18(a). Pitch Rate Response from Elevator Hardover, Flight Condition Two

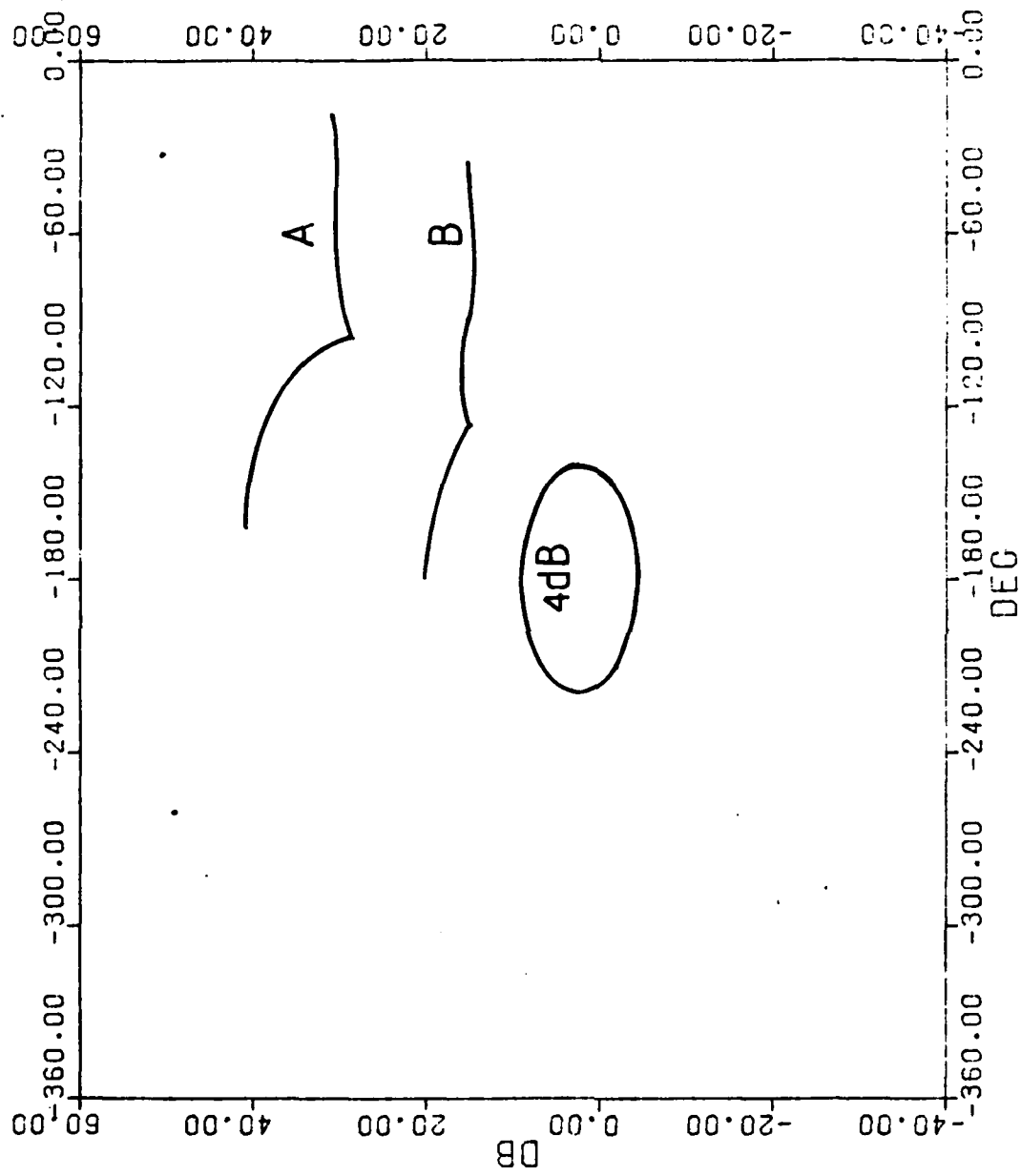


Fig. 5-17. Bounds on L Resulting from  $\tau_{11}$  Demands  
(Bound A =  $\delta_1^a = \delta_1^b \approx 0$ , Bound B =  $\delta_1^a = \delta_1^b \neq 0$ )

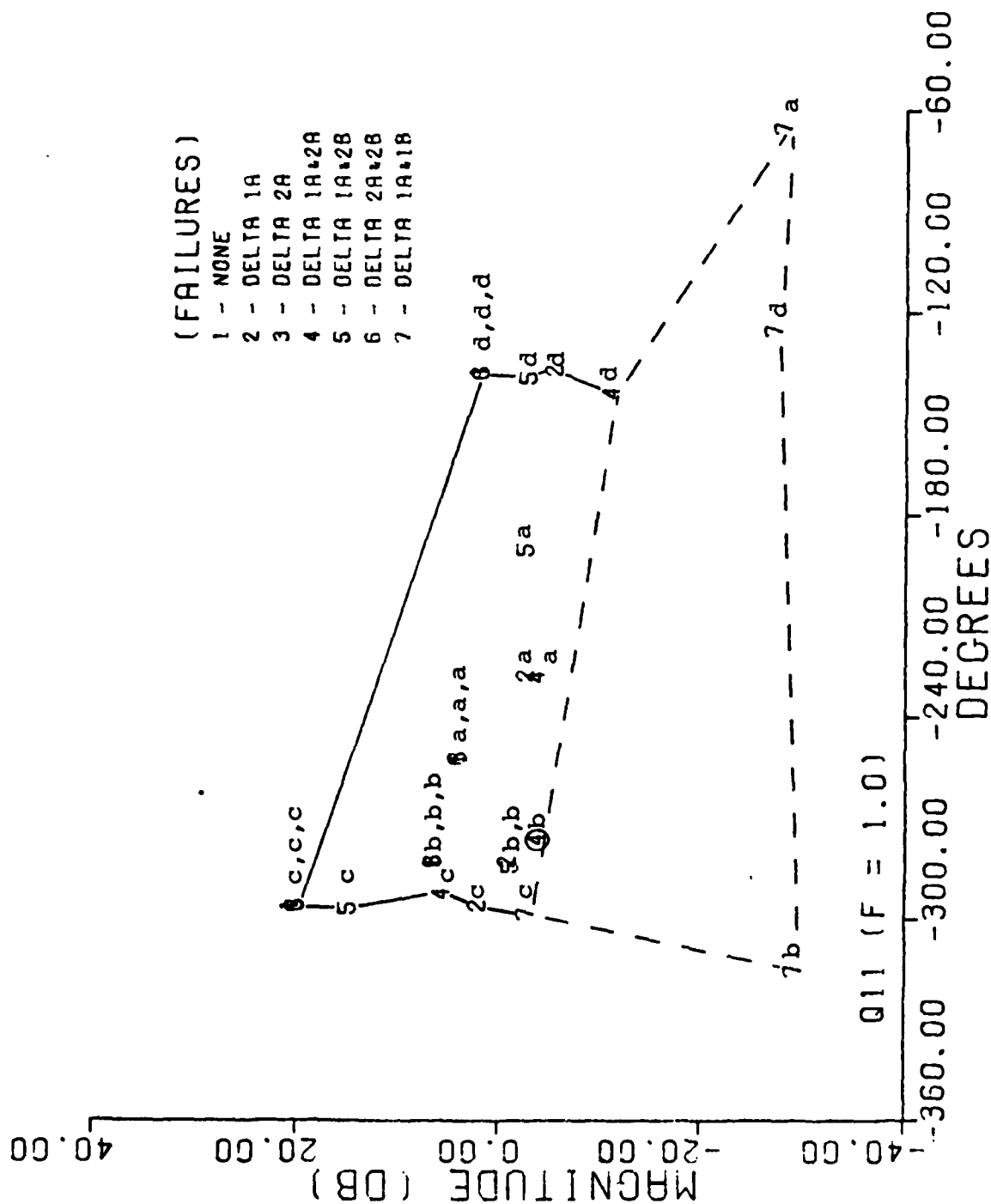


Fig. 5-16. Template of  $Q_{11}$  at  $\omega = 1.0$  r/s, Includes  $\delta_1^a = \delta_1^b = 0$

Figure 5-16. The additional area of plant uncertainty is enclosed by the dashed line. Flight conditions one through four are identified, respectively, by the letters a, b, c, and d. The difference between the bound on L at  $\omega = 1.0$  r/s with and without the plant uncertainty due to both elevators failed is seen in Figure 5-17. The bounds in Figure 5-17 represent the demands on L due to the command response  $\tau_{11}$ . Bound A is developed from the  $Q_{11}$  template with the additional uncertainty of both elevators failed, while bound B is developed from the same template without the extra uncertainty. Clearly, it is seen from Figure 5-17 that the larger the amount of plant uncertainty the higher is the bound on L, and higher bounds mean a larger loop transmission bandwidth. The L bandwidth is increased by more than one and one-half octaves from bound B to bound A.

This example further illustrates the utility of QFT as a design technique for FCS reconfiguration. This method provides the designer with valuable insights very early in the design process as to the "cost" of feedback.

#### V.5 Control Surface "Hardover"

A condition known as "hardover" occurs when a surface without command suddenly deflects to its maximum position and remains there. The output from this control surface may be treated as an external disturbance vector which is to be attenuated by the feedback system. As an

true when the  $p$  input is ramped with a slope of  $50 \text{ deg/sec}^2$  (1 sec to peak value); initial rates are also well below saturation levels as seen in Figure 5-15. Figures 5-14 and 5-15 may be compared with Figures 5-4a and 5-8a where the inputs are pulsed to see the marked reduction in the surface rates.

#### V.4 QFT Insights

Quantitative feedback theory provides the designer with insight at the initial design stages into the nature of the loop transmission. By examining the extent of plant uncertainty, and the narrowness of the performance specifications, the designer gains valuable insight about the bounds on  $L$  and, hence, on  $L$  itself. For instance, the larger the area of plant uncertainty and/or the tighter the performance specifications, the higher are the bounds on  $L$ . Thus, the designer has a feel early in the design for the compensation required. If he feels that the demands on  $L$  are too high he may choose to reduce the amount of plant uncertainty in the design or relax the performance tolerances. For this thesis, it was initially planned that the compensator designs would be robust when both elevators are failed. This was decided against after examining the template of  $Q_{11}$  and seeing the extra "cost" in terms of loop transmission bandwidth for including this configuration. The additional plant uncertainty in the pitch channel when both elevators are failed is seen in

TABLE A-3

AIRCRAFT DATA FOR FLIGHT CONDITION THREE  
(Mach = 0.9 and 20,000 feet)

$$\bar{q} \text{ (dynamic pressure--lbs/ft}^2\text{)} = 552.113$$

$$V_T \text{ (trim velocity--ft/sec)} = 933.23$$

$$\alpha_T \text{ (trim angle-of-attack--deg)} = 1.96$$

Longitudinal Body Axis Primed Dimensional Derivatives

$Z_{\alpha}' = -1.48446$	$M_{\alpha}' = 4.27171$	$X_{\alpha}' = 38.2906$
$Z_{\delta_e}' = -0.149227$	$M_{\delta_e}' = -24.0581$	$X_{\delta_e}' = 2.00593$
$Z_{\delta_f}' = -0.244924$	$M_{\delta_f}' = -6.47269$	$X_{\delta_f}' = 2.31681$
$Z_q' = 0.994789$	$M_q' = -0.777221$	$X_q' = -30.1376$
$Z_u' = -0.000022$	$M_u' = -0.000130$	$X_u' = -0.012075$
$Z_{\theta}' = -0.001120$	$M_{\theta}' = 0.000309$	$X_{\theta}' = -32.1830$

Lateral-Directional Body Axis Primed Dimensional Derivatives

$N_{\beta}' = 7.2370$	$L_{\beta}' = -55.2526$	$Y_{\beta}' = -0.343554$
$N_p' = -0.023184$	$L_p' = -2.80004$	$Y_p' = 0.032636$
$N_r' = -0.362530$	$L_r' = 0.145674$	$Y_r' = -0.997556$
$N_{\delta_r}' = -5.80890$	$L_{\delta_r}' = 10.3955$	$Y_{\delta_r}' = 0.037032$
$N_{\delta_a}' = -1.25006$	$L_{\delta_a}' = -51.0502$	$Y_{\delta_a}' = -0.001371$
$N_{\delta_{DT}}' = -5.13710$	$L_{\delta_{DT}}' = -50.7290$	$Y_{\delta_{DT}}' = 0.026609$
$N_{\delta_c}' = 5.89254$	$L_{\delta_c}' = 5.53185$	$Y_{\delta_c}' = 0.026734$

TABLE A-4

AIRCRAFT DATA FOR FLIGHT CONDITION FOUR  
(Mach = 1.6 and 30,000 feet)

$$\bar{q} \text{ (dynamic pressure--lbs/ft}^2\text{)} = 1129.31$$

$$V_T \text{ (trim velocity--ft/sec)} = 1591.75$$

$$\alpha_T \text{ (trim angle-of-attack--deg)} = 1.68$$

Longitudinal Body Axis Primed Dimensional Derivatives

$Z'_\alpha = -1.25100$	$M'_\alpha = -43.8012$	$X'_\alpha = 45.1941$
$Z'_{\delta_e} = -0.115337$	$M'_{\delta_e} = -32.8919$	$X'_{\delta_e} = 9.40431$
$Z'_{\delta_f} = -0.075084$	$M'_{\delta_f} = -5.85004$	$X'_{\delta_f} = -13.3021$
$Z'_q = 0.995991$	$M'_q = -0.351737$	$X'_q = -46.5132$
$Z'_u = -0.000002$	$M'_u = 0.000802$	$X'_u = -0.030046$
$Z'_\theta = -0.000593$	$M'_\theta = -0.000075$	$X'_\theta = -32.1861$

Lateral-Directional Body Axis Primed Dimensional Derivatives

$N'_\beta = 7.42422$	$L'_\beta = -100.032$	$Y'_\beta = -0.358564$
$N'_p = -0.005040$	$L'_p = -2.46287$	$Y'_p = 0.029374$
$N'_r = -0.383316$	$L'_r = 0.160287$	$Y'_r = -0.998410$
$N'_{\delta_r} = -3.27086$	$L'_{\delta_r} = 4.96942$	$Y'_{\delta_r} = 0.010313$
$N'_{\delta_a} = -2.37673$	$L'_{\delta_a} = -14.1689$	$Y'_{\delta_a} = 0.006763$
$N'_{\delta_{DT}} = -3.45190$	$L'_{\delta_{DT}} = -46.6078$	$Y'_{\delta_{DT}} = 0.005250$
$N'_{\delta_c} = 9.36290$	$L'_{\delta_c} = 12.0662$	$Y'_{\delta_c} = 0.028790$



## Appendix B: Reconfiguration Theory

This appendix contains quantitative inherent reconfiguration theory developed by Dr. Isaac Horowitz. Reconfiguration synthesis theory is developed for 2 input-2 output, 5 input-3 output, and 10 input-6 output systems utilizing control of individual flight control surfaces. Theory to optimize the division of control surface effort under no failure and failure conditions is also presented.

A QUANTITATIVE INHERENT RECONFIGURATION THEORY  
FOR A CLASS OF SYSTEMS

Isaac Horowitz\*

Summary

In aircraft flight control, most control surfaces are in pairs (elevators, ailerons, canards etc), with each pair normally controlled as a single unit. If a surface fails, the usual approach is to attempt explicit identification and switch-in of compensation prepared for that contingency. In this paper each surface is separately controlled, permitting "inherent reconfiguration", wherein the design is apriori made such that despite one or several simultaneous surface failures, the system still satisfies the original performance tolerance (of course over a smaller dynamic range), with the same original fixed compensation.

Inherent reconfiguration is a natural extension of "Quantitative Feedback Theory". (QFT), wherein the system design is tuned to the plant uncertainty set  $P = \{P\}$ , and to the acceptable system output set  $A$ . In QFT one apriori designs so that the system output is in  $A$  for all  $P$  in  $P$ . Surface failures simply enlarge the set  $P$ . The transparency of QFT enables the designer to readily see the extra "cost of feedback" for this enlargement of  $P$  by inclusion of surface failures.

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\*Cohen Professor of Applied Mathematics, Weizmann Institute of Science, Israel and Department of Electrical and Computer Engineering, University of Colorado, Boulder.

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A QUANTITATIVE INHERENT RECONFIGURATION THEORY  
FOR A CLASS OF SYSTEMS

Isaac Horowitz

1. INTRODUCTION

Consider a system with uncertain constrained multiple input - multiple output (MIMO) part denoted as the plant  $P$  with  $m$  control inputs and  $n$  outputs to be controlled. If  $m > n$  then  $m-n$  degrees of freedom are available for optimization purposes. In an "output-controllable" plant (Horowitz and Shaked 1975), the  $n$  desired outputs uniquely determine the remaining  $n$  free inputs. If  $n > m$ , only  $m$  outputs are independently achievable. Hence, an effective  $n \times n$  plant is normally assumed in uncertain MIMO systems, if the problem is to achieve desired input-output relations despite uncertainty (Horowitz 1979, 1983).

There is an important plant class wherein one or more of the  $n$  plant inputs is actually a pair e.g. elevator, canard and aileron pairs in an aircraft. The elevator pair is usually counted as one control input, because its two parts are tied or driven together to achieve longitudinal mode control, and therefore do not affect the lateral variables such as roll and yaw. Similarly the two ailerons are usually tied or driven differentially for lateral control, and therefore this pair does not affect longitudinal variables such as pitch and normal acceleration. However, the individual elements of each of these pairs do affect both the longitudinal and lateral variables and this fact is indeed used for "reconfiguration" purposes in case of significant control surface damage. Such reconfiguration

requires explicit identification of the failure and the switch-in of standby compensation elements designed for this specific purpose. The challenge (Rubertus 1983, Chandler 1984, Chandler and Potts 1983) treated in this paper is to devise a design theory in which no explicit identification is required. Rather, even under severe control surface failures, the tolerances are to be satisfied automatically, with no need for switching in of new compensations. The same compensation functions are to be adequate under normal (no failure) and under the maximum possible control surface failures. This is denoted here as "inherent reconfiguration". The question of how optimum such a single design is for the variety of failure conditions, is discussed in Section 5. Three configurations of increasing complexity are treated in Sections 2-4.

### 1.1 Synthesis objectives

A linear time-invariant plant model is assumed. (See Section 5 for discussion of nonlinear plants). There are specified tolerances on the closed-loop system responses to commands  $r_i$  in Figure 1, as follows. Let  $y = Tr$ ,  $T = [t_{ij}]$ , and each  $t_{ij}$  has its set of acceptable functions  $T_{ij}$  either in the time or frequency domain. It is particularly convenient to assume "basically noninteracting" (BNIA) tolerances of the form  $|t_{ij}(j\omega)| \leq b_{ij}(\omega)$  for  $i \neq j$  (see Section 5 for discussion). There is also a set of possible external disturbance vectors  $D = \{d\}$  acting on the plant whose effect on the system output is given by  $y_d = T_d d$ ,  $T_d = [t_{d_{ij}}]$ , with tolerances of the form  $|t_{d_{ij}}| \leq b_{d_{ij}}(\omega)$ , giving sets of acceptable disturbance response functions  $T_d$ . These tolerances on the elements of  $T$  and  $T_d$

are to be achieved over the entire set of plant variation (different flight conditions in flight control) and over the maximum possible simultaneous control input failures, by means of a fixed set of compensation functions.

## 2. A TWO-PAIR STRUCTURE

### 2.1 Introduction

The simplest plant for illustrating the theory is shown (in darker lines), together with its compensations, in Figure 1. For example, the  $(\delta_1^a, \delta_1^b)$  pair could be the right and left elevators  $(\delta_2^a, \delta_2^b)$  the right and left ailerons,  $y_1$  the pitch angle,  $y_2$  the roll angle. The effect of each individual  $j$  surface on each  $i$  output is given explicitly by  $p_{ij}^a, p_{ij}^b$  with  $p_{ij}^a = p_{ij}^b$  under normal (no failure) conditions. If  $\delta_2^a, \delta_2^b$  (or  $\delta_1^a, \delta_1^b$ ) have equal values, i.e. the right and left ailerons (or elevators) are deflected equally in the same direction, then  $y_2(\text{roll}) = \sum (\delta_i^a p_{2i}^a - \delta_i^b p_{2i}^b) = 0$ . That is why minus signs are associated with  $p_{12}^b, p_{22}^b$  in Figure 1, and plus signs for all  $p_{1i}^a, p_{1i}^b$ .

If the command inputs  $(r_1, r_2)$  and the plant outputs  $(y_1, y_2)$ , but no internal plant variables can be measured (Horowitz 1982a), then the most general feedback structure has 16 free functions:  $8f_{ij}$  from the two  $r_j$  to the four  $\delta_i$  and  $8m_{ij}$  to the four  $\delta_i$  from the two  $y_j$  in Figure 1. But it is assumed here that in the normal (no failure) case, zero interaction between the lateral and longitudinal outputs is desired i.e.  $t_{ij} = 0$  for  $i \neq j$ , also that  $p_{ij}^a = p_{ij}^b$ . This

accounts for the various symmetries in the  $f_{ij}$ ,  $m_{ij}$  of Figure 1. Thus, if  $r_1 = 0$ ,  $r_2 \neq 0$  the desired  $y_1 = 0$ ,  $y_2 \neq 0$  are achievable by  $\delta_1^a = -\delta_1^b$ ,  $\delta_2^a = -\delta_2^b$ , so opposite signals via  $(m_{12}, -m_{12})$ ,  $(m_{22}, -m_{22})$  ( $f_{12}, -f_{12}$ ), ( $f_{22}, -f_{22}$ ) are needed. However, the same  $(m_{11}, m_{11})$  ( $m_{21}, m_{21}$ ) and  $f_{j1}$  give  $\delta_i^a = \delta_i^b$  if  $y_1 \neq 0$ ,  $y_2 = 0$  are wanted in response to  $r_1 \neq 0$ ,  $r_2 = 0$ . Hence, only 8 free compensating functions are left. Note that Figure 1 is only one of an infinitude of possible canonic structures (Horowitz 1963) exploiting the available freedom. Simple equivalence relations permit any one to be replaced by another structure with no need for redesign.

## 2.2 Development of Synthesis Theory

In Figure 1,  $y = Tr = P = P(Fr - My)$ , so

$$T = [t_{ij}] = (I + PM)^{-1}PF \quad \text{with} \quad (1)$$

$$P = \begin{bmatrix} p_{11}^a & p_{11}^b & p_{12}^a & p_{12}^b \\ p_{21}^a & p_{21}^b & p_{22}^a & p_{22}^b \end{bmatrix}, \quad \delta = [\delta_1^a, \delta_1^b, \delta_2^a, \delta_2^b],$$

$$r = [r_1, r_2]', \quad M = \begin{bmatrix} m_{11} & m_{12} \\ m_{11} & -m_{12} \\ m_{21} & m_{22} \\ m_{21} & -m_{22} \end{bmatrix}, \quad F = \begin{bmatrix} f_{11} & f_{12} \\ f_{11} & -f_{12} \\ f_{21} & f_{22} \\ f_{21} & -f_{22} \end{bmatrix} \quad (2a-e)$$

This gives, with all elements  $f_{ij}, m_{ij}, p_{ij}(s)$  transfer functions,

$$PM = \begin{bmatrix} (p_{11} + p_{12} \frac{m_{21}}{m_{11}})m_{11} & (\Delta p_{12} + \frac{m_{12}}{m_{22}} \Delta p_{11})m_{22} \\ (\Delta p_{21} + \frac{m_{21}}{m_{11}} \Delta p_{22})m_{11} & (p_{22} + p_{21} \frac{m_{12}}{m_{22}})m_{22} \end{bmatrix} \quad (3a)$$

$$PF = \begin{bmatrix} (p_{11} + p_{12} \frac{f_{21}}{f_{11}}) f_{11} & (\Delta p_{12} + \frac{f_{12}}{f_{22}} \Delta p_{11}) f_{22} \\ (\Delta p_{21} + \frac{f_{21}}{f_{11}} \Delta p_{22}) f_{11} & (p_{22} + \frac{f_{12}}{f_{22}} p_{21}) f_{22} \end{bmatrix} \quad (3b)$$

$$p_{ij} = p_{ij}^a + p_{ij}^b, \quad \Delta p_{ij} = p_{ij}^a - p_{ij}^b \quad (3c)$$

Quantitative synthesis theories for satisfying each of the  $n^2$   $t_{ij}$  tolerances of (1) despite the uncertainty, have been developed (Horowitz 1979, 1982b) and applied (Horowitz and Loecher 1981, Horowitz et al 1981, 1982, Betzold et al 1984) to significant examples. These techniques have the merit of reducing the  $n \times n$  MIMO problem into a number of single input - single output (SISO) single-loop problems, whose solutions are guaranteed to solve the original MIMO problem. It would be very desirable to apply this theory to the present problem class (see also Section 2.2.2). For this purpose it is necessary to identify a  $n \times n$  equivalent plant  $P_e$  and compensator  $M_e, F_e$  matrices ( $n=2$  in this section), such that

$$T = (I + PM)^{-1} PF \text{ of Eq(1)} = (I + P_e M_e)^{-1} P_e F_e \quad (4)$$

Also for the BNIA tolerances of Section 1.1,  $M_e$  and  $F_e$  should be diagonal. Examination of (3a-c) reveals this is achieved by setting

$$\frac{m_{21}}{m_{11}} = \frac{f_{21}}{f_{11}} \triangleq \mu_{21}(s), \quad \frac{m_{12}}{m_{22}} = \frac{f_{12}}{f_{22}} \triangleq \mu_{12}(s), \text{ giving} \quad (5a,b)$$

$$P_e = [p_{ije}] = \begin{bmatrix} p_{11} + u_{21}p_{12} & \Delta p_{12} + u_{12}\Delta p_{11} \\ \Delta p_{21} + u_{21}\Delta p_{22} & p_{22} + u_{12}p_{21} \end{bmatrix} \quad (6a)$$

$$M_e = \text{diag}[m_{ii}], \quad F_e = \text{diag}[f_{ii}] \quad (6b,c)$$

The available quantitative design theory is directly applicable to the new equivalent  $P_e, M_e, F_e$  system, including surface failures, as is next seen. Note in (6a) that  $p_{ije} (i \neq j) = 0$  in the no failure case. See Section 2.2.2 for additional strong justification of relations (5a,b).

### 2.2.1 Failure cases

Various kinds of failure are possible. Total failure of  $\delta_1^a$  is defined as  $p_{i1}^a = 0$  for all  $i$ , which may involve new values of the other  $p_{ij}$ , if the aerodynamic coefficients involved in these  $p_{ij}$  are affected by the failure. The design theory easily accommodates this with the new value  $p_{ije} (i \neq j)$  no longer zero, but  $p_{12e} = -u_{12}p_{11}^b$ ,  $p_{21e} = -p_{21}^b$  and  $p_{11e} = p_{11}^b + u_{21}p_{12}$ ,  $p_{22e} = p_{22} + u_{12}p_{21}^b$ . Partial failures likewise result in new values of the  $p_{ije}$ , without changing the form of the MIMO problem. The quantitative feedback theory (QFT) of (Horowitz 1979, 1982b) was devised precisely for such situations of the  $p_{ije}$  assuming different values due to uncertainty. The various failure cases simply give additional elements in the set  $P_e = \{[p_{ije}]\}$  of possible plant matrices. Suppose a fixed compensation design is being made for  $m_1$  representative flight conditions involving different Mach and altitude values and  $m_2$  different failure cases. Then at most



$m_1 m_2$  (instead of  $m_1$ ) sets of  $P_e$  matrix element values constitute the plant uncertainty set  $P_e$ , to which the design theory is applied in precisely the same manner as if there were only  $m_1$  sets. It is even conceivable that the extent of the new uncertainty (measured by the ranges of the individual frequency responses  $\{p_{ije}(j\omega)\}$  at any  $\omega$ ) is not much enlarged by the various failure cases.

"Hardover" is a type of failure in which one or more control surfaces are frozen at some fixed values and can no longer be changed. If  $\delta_1^a$  is so frozen at (say) its maximum value  $\delta_{1x}$ , then let its resulting vector of output components  $(p_{11}^a \delta_{1x}, p_{21}^a \delta_{1x})' = d_e$  be treated as an external disturbance vector, which is to be suitably attenuated by the feedback system, in order to satisfy the system performance tolerances. The effective plant upon which  $d_e$  acts is  $P_e$  of (6a) in which  $p_{11}^a = p_{21}^a = 0$ , because  $\delta_1^a$  is no longer controllable. The synthesis theory can handle simultaneous disturbance attenuation and command input response under plant uncertainty.

#### 2.2.2. Additional justification of constraints (5a,b)

One might argue that (5,ab) may not be optimum relations inasmuch as they are apparently required only so that the specific synthesis techniques of Horowitz (1979, 1982b) may be directly used. It is next shown that they are in fact necessary in order that failures may lead to small and controllable channel interaction despite plant uncertainty. Thus, consider the system transfer matrix  $T$  under  $\delta_1^a$  failure, for which  $p_{11}^a = p_{21}^a = 0$ , giving

$$T = \begin{bmatrix} (f_{11}p_{11}^b + f_{21}p_{12}^b)(1+m_{22}p_{22}^b) & p_{11}p_{22}^b(m_{12}f_{22} - m_{22}f_{12}) - f_{12}p_{11}^b \\ + f_{21}p_{12}^b m_{12}p_{21}^b & \\ p_{12}p_{21}^b(m_{11}f_{21} - f_{11}m_{21}) - f_{21}p_{21}^b & (f_{12}p_{21}^b + f_{22}p_{22}^b)(1+m_{21}p_{12}^b) \\ & + f_{22}p_{22}^b m_{11}p_{11}^b \end{bmatrix} \quad (7)$$

$(1+m_{21}p_{12}^b)(1+m_{12}p_{21}^b+m_{22}p_{22}^b) + m_{11}p_{11}^b(1+m_{22}p_{22}^b)$ . The dominant terms in the numerator of any  $t_{uv}$  involve  $m_{..}p_{..}p_{..}$  products, so small  $t_{uv}$  ( $u \neq v$ ) relative to  $t_{uu}$  require such products disappear in the numerators of  $t_{uv}$  independently of the actual  $p$  values. This is achieved in (7) only by (5a,b), because the other alternative of setting for example  $p_{11}p_{22}^b(m_{12}f_{22} - m_{22}f_{12}) = f_{12}p_{11}^b$  is impractical when there is non-negligible plant uncertainty. It is readily found that (5a,b) suffice for this purpose for  $\delta_1^a$ ,  $\delta_1^b$ ,  $\delta_2^a$ ,  $\delta_2^b$  failures one at a time, or any two at a time.

It is arguable that the same result is achieved in (7) by setting  $m_{ij} = f_{ij} = 0$  for all  $i \neq j$ , which is a special case of (5a,b). However it would then be impossible to achieve  $t_{22}$  under simultaneous failure of  $\delta_2^a$  and  $\delta_2^b$ , or to achieve  $t_{11}$  if both  $\delta_1^a$  and  $\delta_2^a$  failed. Also  $\delta_2^a$ ,  $\delta_2^b$  would not contribute to  $y_1$  when  $r_1 \neq 0$  and  $\delta_1^a$ ,  $\delta_1^b$  would not contribute to  $y_2$  when  $r_2 \neq 0$ . Thus (5a,b) are inherently necessary in any synthesis technique which achieves the objectives in this paper.

### 2.2.3 Division of control effort

(a) No failure case. In order to find the  $\delta$ 's needed to achieve the desired outputs note that  $\delta = Fr - My = (F - MT)r$ . Take any  $r_i \neq 0$  the other  $r_j = 0$  so  $Fr = f^i$  the  $i$ th column of  $F$ , and  $Tr = t^i$  the  $i$ th

column of  $T$ . In the no failure case if  $i=1$ , and if  $p_{ij}^a = p_{ij}^b$  giving  $t_{21} = 0$ , the result is  $\delta_1^a = \delta_1^b = f_{11}^{-m_{11}} t_{11}$  and  $\delta_2^a = \delta_2^b = f_{21}^{-m_{21}} t_{11}$ , so that due to (5a),  $\delta_1^a / \delta_2^a = f_{11} / f_{21} = 1 / \mu_{21}(s)$ . Similarly for  $r_2 \neq 0$ ,  $r_1 = 0$  and  $p_{ij}^a = p_{ij}^b$ , in the no failure case  $\delta_1^a = -\delta_1^b = f_{12}^{-m_{12}} t_{22}$ ,  $\delta_2^a = -\delta_2^b = f_{22}^{-m_{22}} t_{22}$ , so (5b) gives  $\delta_1^a / \delta_2^a = f_{12} / f_{22} = \mu_{12}(s)$ . The designer may choose  $\mu_{21}, \mu_{12}$  to achieve the optimum division of control effort between  $\delta_1$  and  $\delta_2$ . Note that in the traditional older design procedure where the lateral and longitudinal modes are decoupled, there is no such division of control effort -- only the  $\delta_i$  are used to achieve  $y_i$ .

The following appears a reasonable way to select the  $\mu_{ij}$ . Under  $r_1$  command  $t_{11} = p_{11} \delta_1^a + p_{12} \delta_2^a$ . Let  $\delta_2^a = \delta_1^a \phi_{21} p_{11} / p_{12}$ ,  $\phi_{21}$  a constant, so that  $t_{11} = p_{11} \delta_1^a (1 + \phi_{21})$ , in order that  $\delta_1^a, \delta_1^b$  work together to achieve  $t_{11}$ ;  $\phi_{21}$  can possibly be chosen on the basis of the relative saturation levels of  $\delta_1^a, \delta_2^a$ . This gives (see 2.2.3)  $\mu_{21} = \phi_{21} p_{11} / p_{12}$ . Similarly, consideration of  $r_2$  command leads to  $\delta_1^a = \delta_2^a \phi_{12} p_{22} / p_{21}$  ( $r_2 \neq 0, r_1 = 0$ ) =  $\mu_{12} \delta_2^a$ . Of course, the  $p_{ij}$  values change for various flight conditions, but the ratios  $p_{11} / p_{12}, p_{22} / p_{21}$  change much less. A compromise fixed  $p_{ii} / p_{ij}$  ratio can be chosen. Also, if scheduling is used, then the  $\mu_{ij}$  values may also be scheduled.

(b) Control effort division under failure The above equation  $\delta = (f^i - M t^i) r_i$  must be properly used under failure, because it refers to the commanded  $\delta$  values. Hence, if for example  $\delta_1^a$  fails, the above gives the commanded but not actual  $\delta_1^a$  value. Simple equations are obtained for the  $\delta$ 's, but the difficulty is in the

$t_{ij}$  values. If  $\delta_1^a$  fails, then  $t_{12}$ ,  $t_{21}$  are no longer zero so for  $r_2=1$ ,  $r_1=0$

$$\begin{aligned}\delta_2^a &= f_{22} - (m_{21}t_{12} + m_{22}t_{22}) = (f_{22} - m_{22}t_{22}) - \mu_{21}m_{11}t_{12} \\ \delta_2^b &= - (f_{22} - m_{22}t_{22}) - \mu_{21}m_{11}t_{12} \\ \delta_1^b &= - \mu_2 (f_{22} - m_{22}t_{22}) - m_{11}t_{12}\end{aligned}\quad (8)$$

It is readily found (Horowitz 1982b), that

$$\begin{aligned}t_{ij} (i \neq j) &= \frac{t_{jj}p_{ije}}{p_{jje}(1+\ell_i)}, \\ f_{ii} - m_{ii}t_{ii} &= \frac{m_{ii}t_{ii}(1+\ell_j-\gamma)}{\ell_i(1+\ell_j)},\end{aligned}\quad (9)$$

where  $\gamma = p_{12e}p_{21e}/p_{11e}p_{22e}$ ,  $\ell_u = m_{uu}q_{uu}$ ,

$$p_e^{-1} = [1/q_{uv}].$$

The above are actually general results, giving

$$\frac{2\delta_1^b}{\delta_2^a - \delta_2^b} = \frac{-\left[\ell_1 \frac{p_{12e}}{p_{11e}} + \mu_{12}(1+\ell_1-\gamma)\right]}{1+\ell_1-\gamma}\quad (10)$$

which is  $-\mu_{12}$  in the no fail case, because then  $p_{12e} = 0$ . If  $\delta_1^a$  fails,  $p_{11}^a = p_{21}^a = 0$  so (10) gives, recalling (6a),  $-\mu_{12}\mu_{21}p_{12}/(p_{11}^b + \mu_{21}p_{12})$  in the control-frequency range where  $|\ell_i(j\omega)| \gg 1-\gamma$ . For  $r_1=1$ , the result is in general

$$\frac{2\delta_1^b}{\delta_2^a + \delta_2^b} = \frac{1}{\mu_{21}} \left[ 1 + \frac{\mu_{12}\ell_2 p_{21e}}{p_{22e}(1+\ell_2-\gamma)} \right]\quad (11)$$

which is  $1/\mu_{21}$  in the no fail case where  $p_{21e} = 0$ , and

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the inputs  $r_i$  to the systems of Figures 1-3. Suppose that the desired response function set for mode  $x$  is  $T_{ix}$ .  $H$  is designed so that its outputs are  $T_{ix}c_{ix}$  in response to command input number  $x$  of value  $c_x$ . Then the  $r_i$  of the systems in this paper have the values  $r_i = T_{ix}c_{ix}$ .  $H$  has as many input points as the modes  $F$  being considered, and new ones can be added without disturbing the designs of  $F$ ,  $M$  of this paper. The latter point is the principal advantage of this approach, but the optimum under BNIA design may not be optimum for each or any of the various modes. Clearly compromises between these two approaches are feasible with the design theory presented here.

It is also worth noting that the well-known "scheduling" technique is obviously easily accommodated into the synthesis procedure. An advantage of quantitative synthesis is that the designer clearly sees the relations between "cost of feedback" (Horowitz et al 1982) and the range of uncertainty and the tolerances, so the trade-off between scheduling complexity and cost of feedback reduction is highly transparent.

condition for obtaining the equivalent linear set is to solve the nonlinear plant equations backwards - given the plant outputs to find the plant inputs. This means the parameters equivalent to the  $\mu_{ij}$  of this paper must first be chosen i.e. one must first decide how to use the freedom due to the plant having more inputs than outputs.

#### BNIA design

Optimization with respect to the  $\mu_{ij}$  has been previously discussed. The assumption of desired "basically noninteracting closed-loop tolerances" is related to the optimization problem, as follows. In advanced fighter aircraft there are many different operating modes. In mode  $i$ , a specific combination of outputs may be wanted in response to command  $r_i$  (a specific set of  $t_{ji}$  for  $j = 1$  to  $n$ ). In mode  $m$  an entirely different set of  $t_{jm}$  in response to  $r_m$ , may be specified with its set of tolerances. It is conceivable that the optimum use of the available freedom (more plant inputs than outputs), may be significantly different for the different modes. If this is sufficiently important, different designs with conceivably different compensations inside the feedback loops, may be made, and the new systems switched in accordance with the mode desired. The design theory presented here is clearly readily applied to this alternative, as the design technique is not exclusively for BNIA systems (Horowitz and Loecher 1981).

This paper offers an alternative approach to the many-mode design problem. A prefilter  $H$  is added whose output provides

based upon control effort division, have been discussed in the paper. Another way of using the available design freedom, is to minimize the  $P_e$  uncertainty. Thus in (6a),  $P_{11e} = P_{11} + \mu_{21}P_{12}$  with correlations often between the parameters of  $P_{11}$  and  $P_{12}$ . One could consider choosing  $\mu_{21}$  so as to minimize the uncertainty range of  $P_{11e}$  due to the various flight conditions and failures. Other optimality criterion could also be considered, e.g. drag minimization.

Sections 2-4 presented the synthesis theory by application to structures whose complexity ranged from (4 plant inputs - 2 outputs) to (10 plant inputs - 6 outputs). In all of these, two "groups" (longitudinal and lateral for aircraft) were assumed. But it is obvious that there can be as many groups as outputs. Recall also (Section 2.1) that different compensation structures may be used, in particular  $G = M$  in the forward path, and a new  $F'$  prefilter in place of  $F$  with  $GF' = F$  and the outputs of  $-P$  and  $F'$  being the inputs into  $G$ .

#### Nonlinear uncertain plants

Linear time-invariant plant models were used in this paper, permitting use of transform equations. The synthesis theory is readily extended to uncertain nonlinear MIMO systems by means of the "linear plant set equivalence technique" (Horowitz 1976, 1981a,b,c, 1982c, Horowitz et al 1980a,b). In this technique the uncertain nonlinear plant set is replaced by linear time invariant equivalent sets. The solutions (compensations) for the latter problem are guaranteed to work for the original nonlinear problem for the set of command and disturbance inputs for which the equivalence has been developed. A necessary



## 5. DISCUSSION

This paper has presented a unified synthesis procedure for flight control design which a priori takes into account in a precise, quantitative manner many possible combinations of control surface failures, the various flight conditions and external disturbances (gusts etc.). This can all be done by means of a fixed compensation design, providing the equivalent plant matrix  $P_e$  satisfies the usual necessary conditions (Horowitz 1982b). The minimum-phase condition ( $\det P_e$  has no right half-plane zeros) guarantees that any arbitrarily narrow tolerances can be satisfied. If  $P_e$  is nonminimum-phase it is necessary that the performance tolerances are compatible with the constrained loop bandwidths achievable.

It is conceivable that a single fixed compensation design to handle such a large variety of conditions, may require impractically large loop bandwidths. If so, one could consider scheduling for different classes of mach number and altitude, in order to reduce the range of uncertainty. Scheduling is quite reliable, whereas failure identification may sometimes be fairly unreliable.

### Optimization

Since each control surface is assumed separately controllable, the number of plant control inputs exceeds the outputs to be controlled in the no failure condition. The exploitation of this freedom constitutes an optimization problem, which must be solved before the design for uncertainty and failure can proceed. The difficulties involved when optimization is

$m_{33}t_{31} \doteq p_{31e}\phi/p_{33e}$  ,  $\phi = f_{11}-m_{11}t_{11}$ . From the matrix equation  $\delta = (F-MT)r$ , for  $r_1 = 1$  and other  $r_i = 0$  , there result five scalar equations  $\delta_i = \phi\mu_{i1}-(\mu_{i2}m_{22}t_{21}+\mu_{i3}m_{33}t_{31})$ ,  $\mu_{ii} = 1$  for  $i = 1$  to 5. The  $m_{22}t_{21}$  ,  $m_{33}t_{31}$  are replaced by  $p_{21e}\phi/p_{22e}$  ,  $p_{31e}\phi/p_{33e}$  of above, giving four equations for  $j = 2$  to 5:

$$\lambda_{j1} = (\delta/\delta_1)r_1 \doteq \frac{\mu_{j1}p_{22e}p_{33e}-(\mu_{j2}p_{21e}p_{33e}+\mu_{j3}p_{22e}p_{31e})}{p_{22e}p_{33e}-(\mu_{12}p_{21e}p_{33e}+\mu_{13}p_{22e}p_{31e})} \quad (20)$$

The  $p_{uve}$  are of course functions of the  $\mu_{ij}$  as per (18,19), so the above equation relates the  $\mu_{ij}$  to the  $p_{ij}$  and  $\lambda_{j1}$ . Similarly four equations relating  $\lambda_{j2}$  ( $j=1,3,4,5$ ) to the  $\mu_{ij}$  and  $p_{ij}$  are obtained by setting only  $r_2 = 1$ , the others zero, etc. By this means ( $r_i = 1$ , all others zero for  $i = 1$  to 6), a total of 24 equations relating the  $\lambda_{ij}$  to the 24  $\mu_{ij}$  are obtained.

#### 4.2 Number of tolerable failures

At least six independent control inputs are needed to achieve linear control of the six outputs in Figure 3, so any four of the ten control surfaces may fail simultaneously. The design can be apriori made so that the desired performance tolerances on all 36  $t_{ij}$  of T are satisfied for any such four simultaneous failures. Five failures will permit only twenty-five  $t_{ij}$  tolerances satisfied, and again this can be incorporated into the design, etc. - all this to be achieved by a single fixed set of compensators.

As in Sections 2, 3 interaction between the two sets (1-3 and 4-6) is zero in the no fail case. But as in Sections 2, 3, the paired  $\delta_i$  of set 1 ( $\delta_1, \delta_2$  here) participate in achieving set 2 outputs in response to set 2 ( $r_3-r_6$ ) commands, and vice versa. The design procedure is the same as in Sections 2, 3. Once the  $\mu_{ij}$  are chosen, the  $P_e$  set is available by listing all flight conditions and surface failures for which the design is to apply, and proceeding precisely as in any quantitative MIMO synthesis problem.

#### 4.1 Choice of $\mu_{ij}$

Essentially the choice of  $\mu_{ij}$  is an optimization problem - how to use the available freedom (more plant inputs than outputs) for each  $r_i$  input. There are 30 each  $m_{ij}, f_{ij}$  to be chosen: Setting  $m_{ij}/m_{jj} = f_{ij}/f_{jj}$  of (20b) gives 24 relations, the actual  $\mu_{ij}$  values gives 24 more and finally quantitative design leads to six each of diagonal  $m_{ii}, f_{ii}$ . However, quantitative design can proceed only after the  $\mu_{ij}$  of (20b) have been chosen, permitting  $P_e$  of (18,19) to be evaluated for the various flight conditions and surface failure cases. Suppose the  $\lambda_{ij} = (\delta_i/\delta_j)_{r_j \neq 0}$  (all  $r_i = 0$  for  $i \neq j$ ) have been chosen, if optimum division of control effort under no fail is the optimality criterion. Then, as in Section 3.1, the  $\lambda_{ij}$  must be related to the  $\mu_{ij}$  and a procedure similar to that in Section 3.1 can be followed: e.g. for  $r_1 = 1$  (all other  $r_i = 0$ ), only  $t_{11}, t_{21}, t_{31}$  exist. From  $(I+P_e M_e)T = P_e F_e$  and neglecting  $t_{31}$  or  $t_{21}$  cf  $t_{11}$  (justifiable in BNIA systems) and  $|\ell_i| \gg 1$  (highly justifiable in control frequency range), one gets  $m_{22}t_{21} \doteq P_{21e}/P_{22e}$ ,

The  $\alpha_{3i} = 0$  for  $i = 4-6$  in order that  $\delta_3 = 0$  for  $r_4, r_5$  or  $r_6$  commands as it is assumed that  $t_{3i} = 0$  is desired. Similarly,  $\alpha_{6j} = 0$  for  $j = 1-3$  so  $\delta_6 = 0$  for  $r_1, r_2$  or  $r_3$  commands, as  $t_{6j} = 0$  is assumed desired.

$P_e$  is derived and the  $\mu_{ij}$  defined in the same manner as in Sections 2.2, 3 for the reasons given in 2.2, 2.2.2 giving

$$T = (I + P_e M_e)^{-1} P_e F_e, \text{ and}$$

First three

$$\text{columns of } P_e = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} & P_{15} \\ P_{21} & P_{22} & P_{23} & P_{24} & P_{25} \\ P_{31} & P_{32} & P_{33} & P_{34} & P_{35} \\ \Delta P_{41} & \Delta P_{42} & 0 & \Delta P_{44} & \Delta P_{45} \\ \Delta P_{51} & \Delta P_{52} & 0 & \Delta P_{54} & \Delta P_{55} \\ \Delta P_{61} & \Delta P_{62} & 0 & \Delta P_{64} & \Delta P_{65} \end{bmatrix} \quad \begin{bmatrix} 1 & \mu_{12} & \mu_{13} \\ \mu_{21} & 1 & \mu_{23} \\ \mu_{31} & \mu_{32} & 1 \\ \mu_{41} & \mu_{42} & \mu_{43} \\ \mu_{51} & \mu_{52} & \mu_{53} \end{bmatrix} \quad (18)$$

Last three

$$\text{columns of } P_e = \begin{bmatrix} \Delta P_{11} & \Delta P_{12} & \Delta P_{14} & \Delta P_{15} & 0 \\ \Delta P_{21} & \Delta P_{22} & \Delta P_{24} & \Delta P_{25} & 0 \\ \Delta P_{31} & \Delta P_{32} & \Delta P_{34} & \Delta P_{35} & 0 \\ P_{41} & P_{42} & P_{44} & P_{45} & P_{46} \\ P_{51} & P_{52} & P_{54} & P_{55} & P_{56} \\ P_{61} & P_{62} & P_{64} & P_{65} & P_{66} \end{bmatrix} \quad \begin{bmatrix} \mu_{14} & \mu_{15} & \mu_{16} \\ \mu_{24} & \mu_{25} & \mu_{26} \\ 1 & \mu_{45} & \mu_{46} \\ \mu_{54} & 1 & \mu_{56} \\ \mu_{64} & \mu_{65} & 1 \end{bmatrix} \quad (19)$$

$$\Delta p_{ij} = p_{ij}^a - p_{ij}^h \quad (j \neq 3, 6), \quad \mu_{ij} = \frac{m_{ij}}{m_{jj}} = \frac{f_{ij}}{f_{jj}},$$

$$p_{ij} = p_{ij}^a + p_{ij}^b \quad (j \neq 3, 6). \quad (20a-e)$$

$$M_e = \text{diag. } m_{ii}, \quad F_e = \text{diag. } f_{ii}$$

situations can be included in the design.

#### 4. TEN INPUT - SIX OUTPUT PLANT

The final structure treated here (Figure 3) has one single and two pairs of control surfaces for one set of variables (e.g. longitudinal) and a similar number for a second set (e.g. lateral). The resulting P,F,M matrices are

$$P = \begin{bmatrix} p_{11}^a & p_{11}^b & p_{12}^a & p_{12}^b & p_{13} & p_{14}^a & p_{14}^b & p_{15}^a & p_{15}^b & 0 \\ p_{21}^a & p_{21}^b & p_{22}^a & p_{22}^b & p_{23} & p_{24}^a & p_{24}^b & p_{25}^a & p_{25}^b & 0 \\ p_{31}^a & p_{31}^b & p_{32}^a & p_{32}^b & p_{33} & p_{34}^a & p_{34}^b & p_{35}^a & p_{35}^b & 0 \\ p_{41}^a & -p_{41}^b & p_{42}^a & -p_{42}^b & 0 & p_{44}^a & -p_{44}^b & p_{45}^a & -p_{45}^b & p_{46} \\ p_{51}^a & -p_{51}^b & p_{52}^a & -p_{52}^b & 0 & p_{54}^a & -p_{54}^b & p_{55}^a & -p_{55}^b & p_{56} \\ p_{61}^a & -p_{61}^b & p_{62}^a & -p_{62}^b & 0 & p_{64}^a & -p_{64}^b & p_{65}^a & -p_{65}^b & p_{66} \end{bmatrix} \quad (16)$$

It is assumed  $\delta_6$  affects only the second set of outputs and  $\delta_3$  only the first set, hence  $p_{i6} = 0$  for  $i = 1-3$ ,  $p_{j3} = 0$  for  $j = 4-6$ .

$$M, F = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} & \alpha_{15} & \alpha_{16} \\ \alpha_{11} & \alpha_{12} & \alpha_{13} & -\alpha_{14} & -\alpha_{15} & -\alpha_{16} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} & \alpha_{25} & \alpha_{26} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & -\alpha_{24} & -\alpha_{25} & -\alpha_{26} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & 0 & 0 & 0 \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} & \alpha_{45} & \alpha_{46} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & -\alpha_{44} & -\alpha_{45} & -\alpha_{46} \\ \alpha_{51} & \alpha_{52} & \alpha_{53} & \alpha_{54} & \alpha_{55} & \alpha_{56} \\ \alpha_{51} & \alpha_{52} & \alpha_{53} & -\alpha_{54} & -\alpha_{55} & -\alpha_{56} \\ 0 & 0 & 0 & \alpha_{64} & \alpha_{65} & \alpha_{66} \end{bmatrix} \quad (17)$$

done as follows. As noted in 2.2.3, in general in Figures 1,2  
 $\delta = (F-MT)r$ , giving for  $r_1$  command in Figure 2:

$$\delta_1^a = f_{11} - (m_{11}t_{11} + m_{12}t_{21}),$$

$$\delta_2 = f_{21} - (m_{21}t_{11} + m_{22}t_{21}) = \mu_{21}(f_{11} - m_{11}t_{11}) - m_{22}t_{21}$$

$$\delta_3^a = f_{31} - (m_{31}t_{11} + m_{32}t_{21}) = \mu_{31}(f_{11} - m_{11}t_{11}) - \mu_{32}m_{22}t_{21}.$$

Equations (9) are used to eliminate  $t_{21}$ ,  $f_{11} - m_{11}t_{11}$ .

The result, after writing the  $P_{ije}$  in terms of  $p_{uv}$  and  $\mu_{uv}$  of (13), is

$$\begin{aligned} x_1(\lambda_{21}p_{22} + p_{21}) + x_2p_{23} + x_3\lambda_{21}p_{23} - \frac{\mu_{21}}{m_{22}} &= \frac{-\lambda_{21}}{m_{22}} \\ x_1p_{22}\lambda_{31} - x_2p_{22} + x_3(\lambda_{31}p_{23} + p_{21}) - \frac{\mu_{31}}{m_{22}} &= \frac{-\lambda_{31}}{m_{22}} \quad (15) \\ x_1 = 1 - \mu_{12}\mu_{21}, \quad x_2 = \mu_{31} - \mu_{21}\mu_{32}, \quad x_3 = \mu_{32} - \mu_{12}\mu_{31} \end{aligned}$$

Two more equations are obtained for  $r_2 \neq 0$ ,  $r_1 = 0$ , setting  $\delta_1^a/\delta_2 = \lambda_{12}$ ,  $\delta_3^a/\delta_2 = \lambda_{32}$  giving a total of four equations in the four  $\mu_{ij}$  unknowns. Unfortunately the as yet unknown  $m_{22}$  appears in the first two and  $m_{11}$  in the last two, but the terms containing these  $m_{ii}$  appear small compared to the others, so the result should be relatively insensitive to the  $m_{ii}$  values.

### 3.2 Number of tolerable failures

It is readily seen that there exists a linear range of control of all three outputs in Figure 2, even if any two (of the five) surfaces totally fail simultaneously. But failure of any three leaves only two outputs or any two independent combinations, independently controllable, while failure of four leaves only one output controllable. All these

inherently necessary for it to be possible to achieve under failure longitudinal output with small lateral effect, and vice versa.

### 3.1 Division of control effort and choice of $\mu_{ij}$

One is tempted to follow the procedure in Section 2.2.3, if optimum control effort division is desired i.e. for  $r_1 \neq 1$ ,  $r_2 = r_3 = 0$  in  $t_{11} = p_{11}\delta_1^a + p_{12}\delta_2^a + p_{13}\delta_3^a$ , let  $\delta_2/\delta_1^a = \phi_{21}p_{11}/p_{12}$ ,  $\delta_3/\delta_1^a = \phi_{31}p_{11}/p_{13}$  with  $\phi_{21}, \phi_{31}$  constants, in order that the three  $\delta$ 's work together to achieve  $t_{11}$ . However, the resulting  $t_{21} = p_{11}\delta_1^a(1+\phi_{21}+\phi_{31})$  is then not independently realizable, so the above cannot be followed for  $r_1, r_2$  commands. It can be applied to the  $r_3$  command because  $t_{13} = t_{23} = 0$ ,  $t_{33} = p_{31}\delta_1^a + p_{33}\delta_3^a$ . Let  $\delta_1^a/\delta_3^a = \phi_{13}p_{33}/p_{31}$ . Also  $\delta_1^a = f_{13} - m_{13}t_{33} = \mu_{13}(f_{33} - m_{33}t_{33})$ ,  $\delta_3^a = f_{33} - m_{33}t_{33}$  so  $\delta_1^a/\delta_3^a = \mu_{13}$  which gives  $\mu_{13} = \phi_{13}p_{33}/p_{31}$  once the optimum choice of  $\delta_1^a/\delta_3^a$  has been made. But for  $r_1, r_2$  commands the optimal division of the control effort is more complicated as it is subject to the constraints of  $t_{21}, t_{12}$  satisfying their BNIA tolerances of the form  $|t_{21}(j\omega)/t_{11}(j\omega)| \leq \epsilon_{21}(\omega)$ ,  $|t_{12}, t_{22}| \leq \epsilon_{12}(\omega)$  with  $\epsilon_{21}, \epsilon_{12}$  known. Thus if  $t_{11}/t_{21} = \Sigma p_{1i}\delta_i^a / \Sigma p_{2i}\delta_i^a = 1/\epsilon_{21}$ , with  $\epsilon_{21}$  very small, this gives the constraint  $p_{21} + p_{22}\lambda_{21} + p_{23}\lambda_{31} \neq 0$  where  $\lambda_{i1} = (\delta_i/\delta_1)r_1 \neq 0$ ; for  $r_2$  command the resulting constraint due to  $t_{12}$  is  $p_{11}\lambda_{12} + p_{12} + p_{13}\lambda_{32} \neq 0$ ,  $\lambda_{i2} = (\delta_i/\delta_2)r_2 \neq 0$ .

Suppose the  $\lambda_{ij}$  values have been "optimally" chosen, it is then necessary to relate them to the  $\mu_{ij}$  of (14-c) in order for  $P_e$  of (13) to be known and the design to proceed. This can be

$$P = \begin{bmatrix} p_{11}^a & p_{11}^b & p_{12}^a & p_{13}^a & p_{13}^b \\ p_{21}^a & p_{21}^b & p_{22}^a & p_{23}^a & p_{23}^b \\ p_{31}^a & -p_{31}^a & 0 & p_{33}^a & -p_{33}^b \end{bmatrix} ; M, F = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{11} & \alpha_{12} & -\alpha_{13} \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \\ \alpha_{31} & \alpha_{32} & -\alpha_{33} \end{bmatrix} \quad (12a,b)$$

$\alpha = m$  for  $M$ ,  $\alpha = f$  for  $F$ .

Note that  $p_{32} = 0$  as the rudder has no effect on the longitudinal output;  $\alpha_{23} = 0$  as  $\delta_2 = 0$  is desired for  $r_3$  command giving  $y_3$  output with desired  $y_1 = y_2 = 0$ ; as before  $p_{ij}^a = p_{ij}^b$  is assumed.

As in Section 2.2, one demands  $T = (I + P_e M_e)^{-1} P_e F_e = (I + PM)^{-1} PF$  with  $3 \times 3$   $P_e$  and diagonal  $M_e, F_e$  giving for the analogs of (5,6)

$$P_e = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ \Delta p_{31} & 0 & \Delta p_{33} \end{bmatrix} \quad \begin{bmatrix} 1 & \mu_{12} \\ \mu_{21} & 1 \\ \mu_{31} & \mu_{32} \end{bmatrix} \quad \text{for its first 2 columns,} \quad (13)$$

$$\begin{bmatrix} \Delta p_{11} & \Delta p_{13} \\ \Delta p_{21} & \Delta p_{23} \\ p_{31} & p_{33} \end{bmatrix} \quad \begin{bmatrix} \mu_{13} \\ 1 \end{bmatrix} \quad \text{for its third column}$$

$$\Delta p_{ij} = p_{ij}^a - p_{ij}^b, \quad p_{ij} = p_{ij}^a + p_{ij}^b, \quad \mu_{ij} = \frac{m_{ij}}{m_{jj}} = \frac{f_{ij}}{f_{jj}} \quad (14a-e)$$

$$M_e = \text{diag. } m_{ii}, \quad F_e = \text{diag. } f_{ii}$$

Note that  $p_{31e}, p_{32e}, p_{13e}, p_{23e} = 0$  in the no fail case.

The results in Sections 2.2.1, 2.2.2 apply as again (14c) is



be incorporated in the design, to guarantee the desired performance under these circumstances.

#### 2.2.5 Design procedure

The plant of Figure 1 has four control inputs (the four  $\delta$ s) and only two outputs, so there exist two degrees of freedom ( $\mu_{12}, \mu_{21}$ ) available for optimization. It was shown in Section 2.2.3 how these may be chosen for optimum division of the control effort. However other optimal criteria may be preferable (see Section 5). In any case, it is assumed that  $\mu_{12}, \mu_{21}$  have been chosen, which determines  $P_e$ . The various modes and conditions for which the system is to operate are listed by the designer, the flight conditions, the failure cases, the disturbances (gusts etc) etc. These give the sets  $P_e$ ,  $D$  of Sections 1.1, 2.2.1 and the performance tolerances  $T$ ,  $T_d$  of Section 1.1. The design technique of (Horowitz 1979, 1982b) is then applied, resulting in required  $m_{ii}(s)$ ,  $f_{ii}(s)$ ,  $i=1,2$ . The values of the  $m_{ij}, f_{ij}$  for  $i \neq j$  are available from (5a,b) and the design is complete.

### 3. A FIVE INPUT - THREE OUTPUT PLANT

The slight increase in plant complexity in Figure 2 results in a more difficult problem of  $\mu_{ij}$  selection, if control effort division is the optimality criterion. The added single control input  $\delta_2$  and the pair  $(\delta_1^a, \delta_2^a)$  are assumed all of one type say lateral e.g.  $\delta_2$  is a rudder,  $(\delta_1^a, \delta_1^b)$  is an aileron pair while  $(\delta_3^a, \delta_3^b)$  is a longitudinal type, e.g. elevator pair. If so the  $P, M, F$  matrices are

$\approx p_{22}/\mu_{21}(p_{22}+\mu_{12}p_{21}^b)$  in the control frequency range where  $|\ell_2(j\omega)| \gg 1-\gamma$ . Equations (10,11) can be used to obtain the control effort ratios for other failure cases.

Thus, if  $\mu_{12}, \mu_{21}$  provide the optimum division of control efforts between  $\delta_1$  and  $\delta_2$  in the no fail case, then in view of the above, the division under failure may no longer be optimum. But in failure cases, the dominating concern is acceptable performance, giving more time for reliable identification and switching in of compensations optimum for the failure situation, should this be considered justifiable.

#### 2.2.4 Performance under failures

The quantitative MIMO synthesis techniques (Horowitz 1979, 1982b) guarantee the achievement, theoretically, of arbitrarily narrow performance tolerances for a large class of uncertain MIMO plants. The design may not be practical if the sensors are not sufficiently accurate or have unduly large noise levels. The principal condition for this theoretical ability is that the MIMO plant is minimum-phase over its range of uncertainty i.e.  $\det. P_e$  has no right half-plane zeros and that  $P_e$  is output controllable (Horowitz and Shaked 1975, Horowitz 1982b) meaning that despite the failures, the remaining control surfaces still have the physical ability to achieve two independent outputs (of course for a smaller output range). This is so in the present problem for any single, or any two simultaneous control surface failures. If there are three simultaneous failures, then only one output is achievable and the second output is not independently achievable. Such cases too can

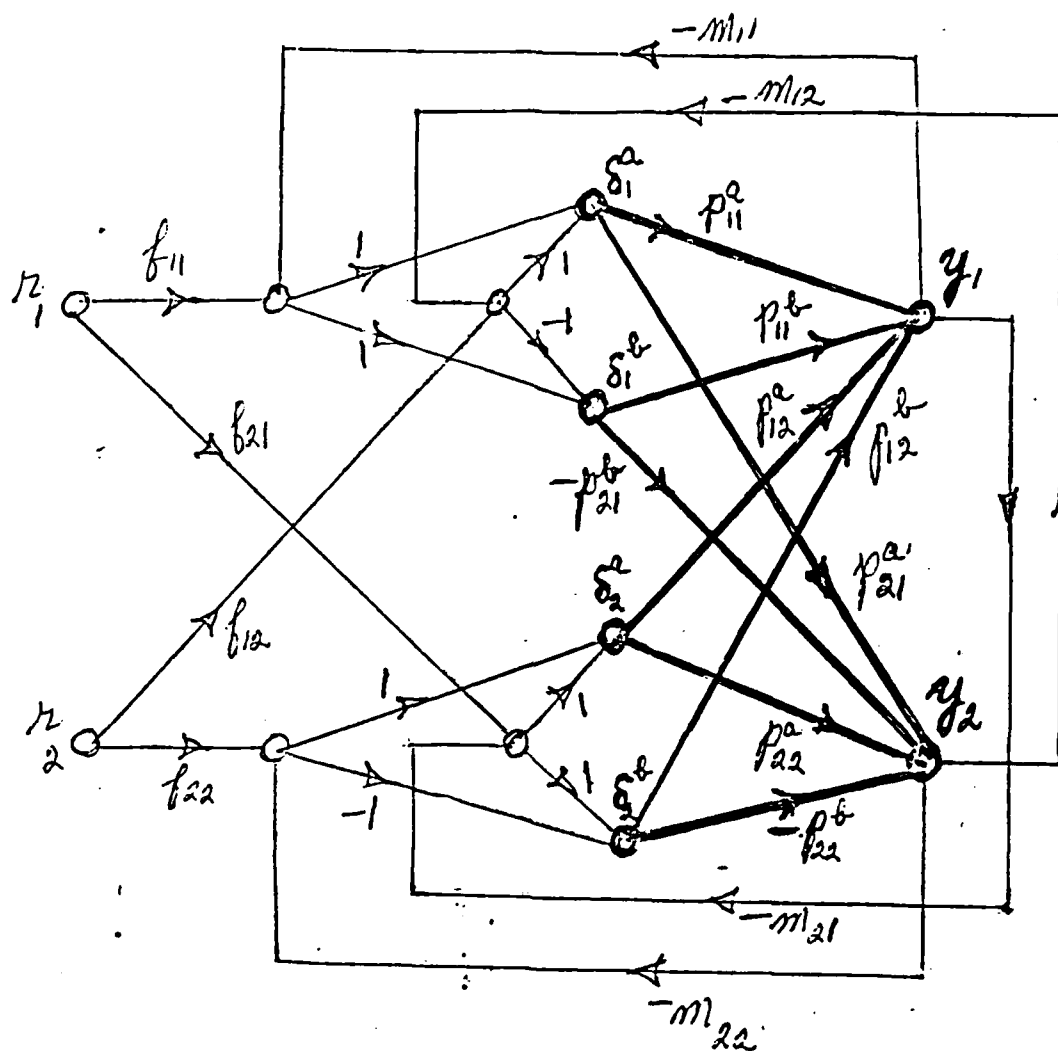


Fig. B-1. A 4-input, 2-output, 2 mode structure.

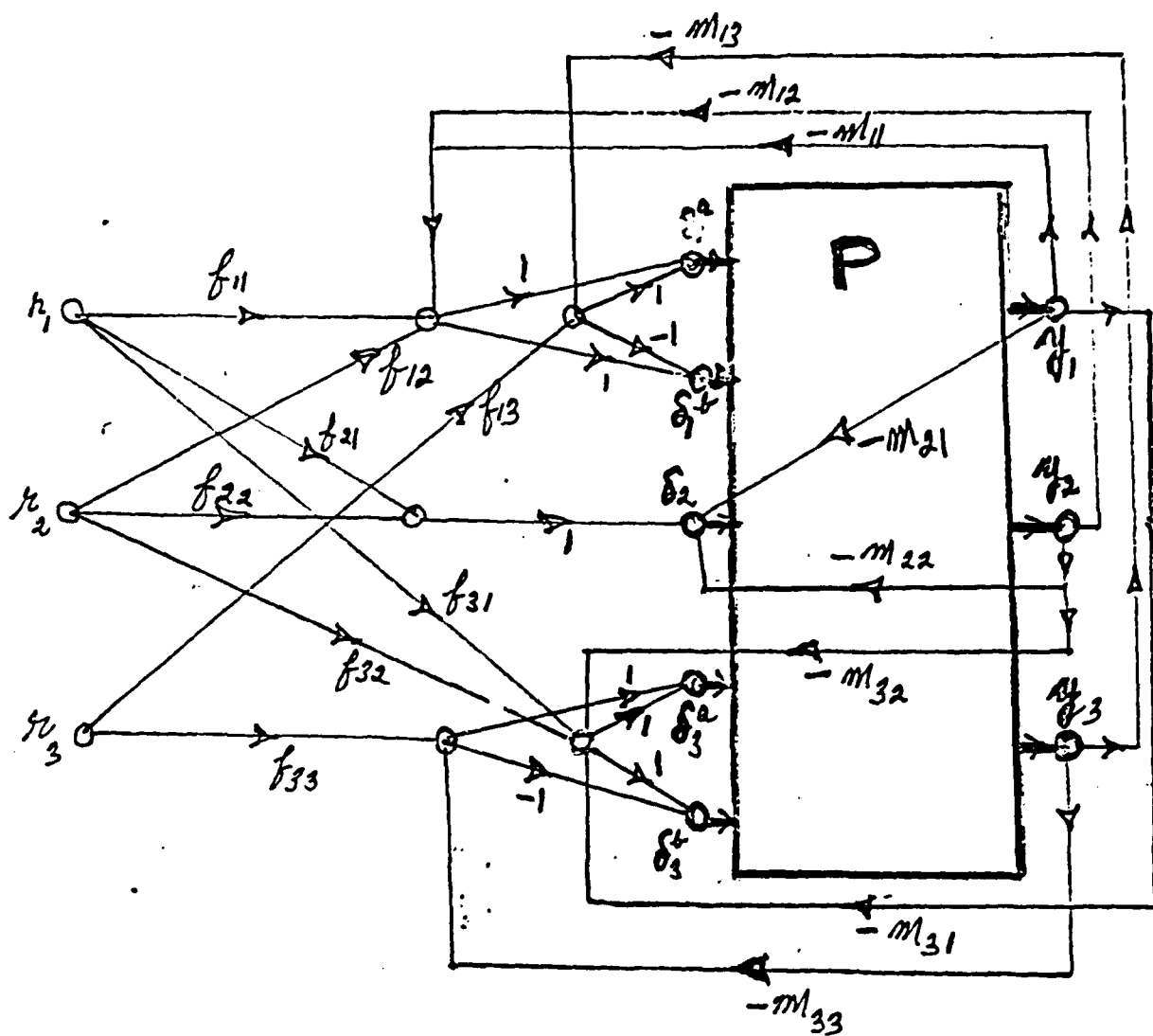


Fig. B-2. A 5-input, 3-output, 2-mode structure.

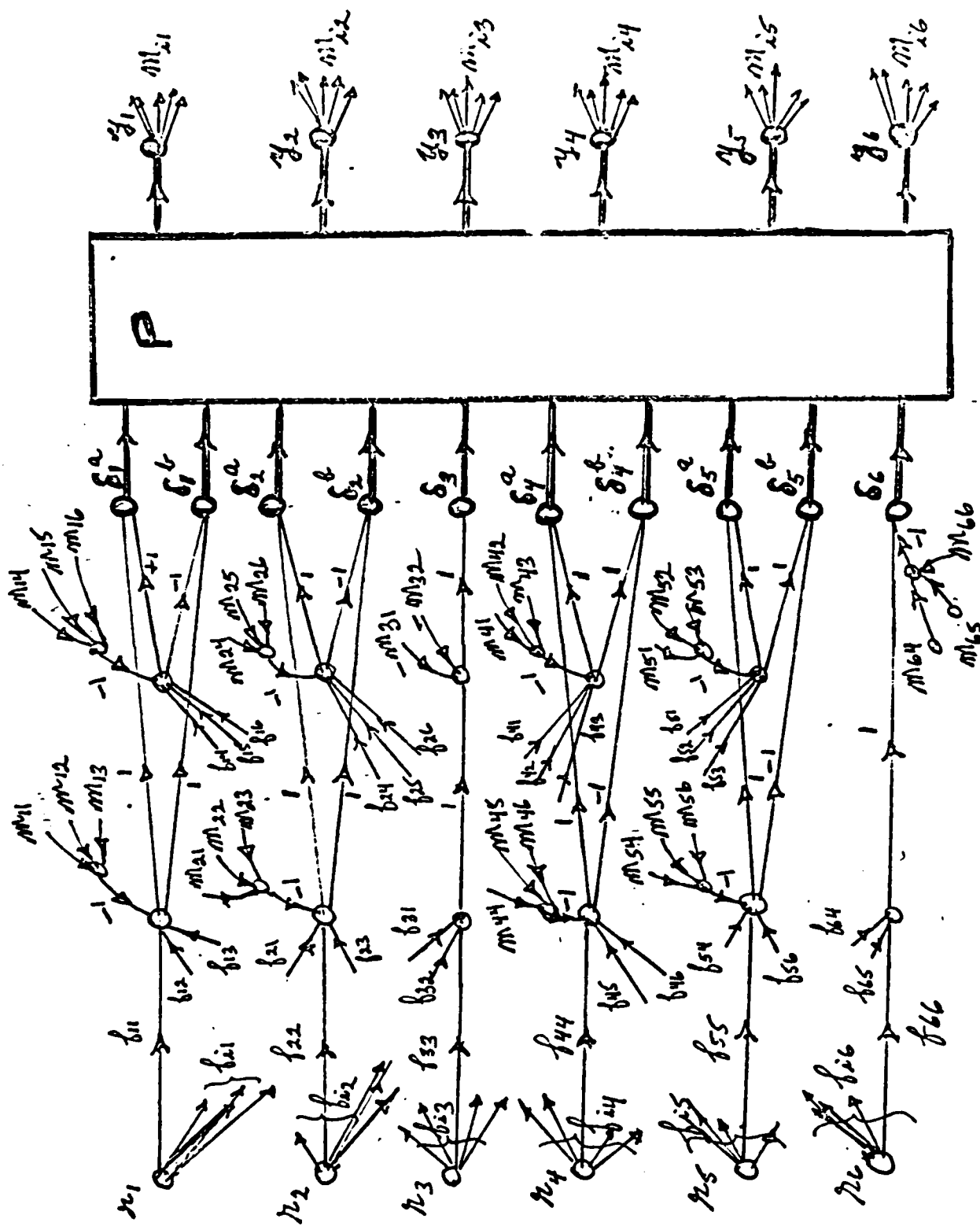


Fig. B-3. A 10-input, 6-output, 2-mode structure.

### Appendix C: Equivalent Plant Transfer Function Matrices

The equivalent plant transfer function matrices are listed in this appendix for each flight condition and control surface configuration mode. These matrices are derived from the  $\underline{P}_e$  matrix below using the basic plant transfer function equations developed in Chapter II.

$$\underline{P}_e = \begin{bmatrix} P_{11} + \mu P_{12} & \Delta P_{12} + \mu \Delta P_{11} \\ \Delta P_{21} + \mu \Delta P_{22} & P_{22} + \mu P_{21} \end{bmatrix}$$

where

$$P_{ij} = P_{ij}^a + P_{ij}^b \text{ and } \Delta P_{ij} = P_{ij}^a - P_{ij}^b$$

The reciprocal plant expressions ( $1/Q_{ij}$ ) for the SISO systems of Figure 4-5 are obtained by inverting the  $\underline{P}_e$  matrices. These transfer functions are expressed in terms of the  $Q_{ij}$  equations in this appendix.  $\underline{P}_e^{-1} = (1/Q_{ij})$ .

TABLE C-1

## EQUIVALENT PLANT MATRICES FOR FLIGHT CONDITION ONE

<u>Mode</u> <u>1:</u>	$\underline{P_{11e}^*}$ -2.176 (0) (-.0175) (-.4617) (.363) (-1.3) (-.0768±j.207)	$\underline{P_{12e}^*}$ --
	$\underline{P_{21e}^*}$ --	$\underline{P_{22e}^*}$ -5.549 (0) (-.2237±j1.029) (-.104) (-.684) (-.274±j1.91)
<u>Mode</u> <u>2:</u>	$\underline{P_{11e}}$ -1.058 (0) (-.01677) (-.4669) (.363) (-1.3) (-.0768±j.207)	$\underline{P_{12e}}$ .2795 (0) (-.01822) (-.4568) (.363) (-1.3) (-.0768±j.207)
	$\underline{P_{21e}}$ 2.142 (0) (-.3017±j1.562) (-.104) (-.684) (-.274±j1.91)	$\underline{P_{22e}}$ -5.041 (0) (-.2153±j.9547) (-.104) (-.684) (-.274±j1.91)
<u>Mode</u> <u>3:</u>	$\underline{P_{11e}}$ -2.206 (0) (-.01787) (-.4592) (.363) (-1.3) (-.0768±j.207)	$\underline{P_{12e}}$ -.121 (0) (-.0654) (-.259) (.363) (-1.3) (-.0768±j.207)
	$\underline{P_{21e}}$ .56 (0) (-.21±j.853) (-.104) (-.684) (-.274±j1.91)	$\underline{P_{22e}}$ -3.3 (0) (-.2363±j1.13) (-.104) (-.684) (-.274±j1.91)

\*Equivalent Plant.

TABLE C-1--Continued

<u>Mode</u> <u>4:</u>	<u>P<sub>11e</sub></u>	<u>P<sub>12e</sub></u>
	$\frac{-1.09(0) (-.0175) (-.462)}{(.363) (-1.3) (-.0768 \pm j.207)}$	$\frac{.159(0) (-.003) (-.587)}{(.363) (-1.3) (-.0768 \pm j.207)}$
	<u>P<sub>21e</sub></u>	<u>P<sub>22e</sub></u>
	$\frac{2.7(0) (-.282 \pm j1.45)}{(-.104) (-.684) (-.274 \pm j1.91)}$	$\frac{-2.78(0) (-.224 \pm j1.03)}{(-.104) (-.684) (-.274 \pm j1.91)}$
<u>Mode</u> <u>5:</u>	<u>P<sub>11e</sub></u>	<u>P<sub>12e</sub></u>
	$\frac{-1.09(0) (-.0175) (-.462)}{(.363) (-1.3) (-.0768 \pm j.207)}$	$\frac{.4(0) (-.027) (-.402)}{(.363) (-1.3) (-.0768 \pm j.207)}$
	<u>P<sub>21e</sub></u>	<u>P<sub>22e</sub></u>
	$\frac{1.582(0) (-.336 \pm j1.74)}{(-.104) (-.684) (-.274 \pm j1.91)}$	$\frac{-2.78(0) (-.224 \pm j1.03)}{(-.104) (-.684) (-.274 \pm j1.91)}$
<u>Mode</u> <u>6:</u>	<u>P<sub>11e</sub></u>	<u>P<sub>12e</sub></u>
	$\frac{-2.236(0) (-.0182) (-.457)}{(.363) (-1.3) (-.0768 \pm j.207)}$	--
	<u>P<sub>21e</sub></u>	<u>P<sub>22e</sub></u>
	--	$\frac{-1.07(0) (-.302 \pm j1.56)}{(-.104) (-.684) (-.274 \pm j1.91)}$



TABLE C-2

## EQUIVALENT PLANT MATRICES FOR FLIGHT CONDITION TWO

<u>Mode</u> <u>1:</u>	<u>P<sub>11e</sub>*</u>	<u>P<sub>12e</sub>*</u>
	$\frac{-5.92(0) (-.0099) (-.562)}{(1.167) (-2.02) (-.0065 \pm j.078)}$	--
	<u>P<sub>21e</sub>*</u>	<u>P<sub>22e</sub>*</u>
	--	$\frac{-20.8(0) (-.223 \pm j1.697)}{(.078) (-.827) (-.211 \pm j1.95)}$
<u>Mode</u> <u>2:</u>	<u>P<sub>11e</sub></u>	<u>P<sub>12e</sub></u>
	$\frac{-2.98(0) (-.0098) (-.574)}{(1.167) (-2.02) (-.0065 \pm j.078)}$	$\frac{.73(0) (-.01) (-.55)}{(1.167) (-2.02) (-.0065 \pm j.078)}$
	<u>P<sub>21e</sub></u>	<u>P<sub>22e</sub></u>
	$\frac{6.79(0) (-.244 \pm j2.101)}{(-.078) (-.827) (-.211 \pm j1.95)}$	$\frac{-19.14(0) (-.221 \pm j1.66)}{(.078) (-.827) (-.211 \pm j1.95)}$
<u>Mode</u> <u>3:</u>	<u>P<sub>11e</sub></u>	<u>P<sub>12e</sub></u>
	$\frac{-5.89(0) (-.01) (-.56)}{(1.167) (-2.02) (-.0065 \pm j.078)}$	$\frac{.106(0) (-.0067) (-1.86)}{(1.167) (-2.02) (-.0065 \pm j.078)}$
	<u>P<sub>21e</sub></u>	<u>P<sub>22e</sub></u>
	$\frac{2.18(0) (-.219 \pm j1.61)}{(-.078) (-.827) (-.211 \pm j1.95)}$	$\frac{-12.12(0) (-.22 \pm j1.7)}{(.078) (-.827) (-.211 \pm j1.95)}$

\*Equivalent Plant.

TABLE C-2--Continued

<u>Mode</u> <u>4:</u>	<u>P<sub>11e</sub></u>	<u>P<sub>12e</sub></u>
	$\frac{-2.96(0) (-.01) (-.56)}{(1.167) (-2.02) (-.0065 \pm j.078)}$	$\frac{.84(0) (-.01) (-.72)}{(1.167) (-2.02) (-.0065 \pm j.078)}$
	<u>P<sub>21e</sub></u>	<u>P<sub>22e</sub></u>
	$\frac{8.97(0) (-.24 \pm j1.99)}{(-.078) (-.827) (-.211 \pm j1.95)}$	$\frac{-10.4(0) (-.22 \pm j1.7)}{(.078) (-.827) (-.211 \pm j1.95)}$
<u>Mode</u> <u>5:</u>	<u>P<sub>11e</sub></u>	<u>P<sub>12e</sub></u>
	$\frac{-2.96(0) (-.01) (-.56)}{(1.167) (-2.02) (-.0065 \pm j.078)}$	$\frac{.63(0) (-.013) (-.326)}{(1.167) (-2.02) (-.0065 \pm j.078)}$
	<u>P<sub>21e</sub></u>	<u>P<sub>22e</sub></u>
	$\frac{4.6(0) (-.256 \pm j2.3)}{(-.078) (-.827) (-.211 \pm j1.95)}$	$\frac{-10.4(0) (-.22 \pm j1.7)}{(.078) (-.827) (-.211 \pm j1.95)}$
<u>Mode</u> <u>6:</u>	<u>P<sub>11e</sub></u>	<u>P<sub>12e</sub></u>
	$\frac{-2.93(0) (-.01) (-.55)}{(1.167) (-2.02) (-.0065 \pm j.078)}$	—
	<u>P<sub>21e</sub></u>	<u>P<sub>22e</sub></u>
	—	$\frac{-1.7(0) (-.244 \pm j2.1)}{(.078) (-.827) (-.211 \pm j1.95)}$

TABLE C-3

## EQUIVALENT PLANT MATRICES FOR FLIGHT CONDITION THREE

<u>Mode</u> <u>1:</u>	<u>P<sub>11e</sub></u> <sup>*</sup> -25.7 (0) (-.013) (-1.52) (.965) (-3.22) (-.0076±j.538)	<u>P<sub>12e</sub></u> <sup>*</sup> --
	<u>P<sub>21e</sub></u> <sup>*</sup> --	<u>P<sub>22e</sub></u> <sup>*</sup> -63.7 (0) (-.358±j3.07) (-.0272 (-2.7) (-.39±j2.96)
<u>Mode</u> <u>2:</u>	<u>P<sub>11e</sub></u> -13.7 (0) (-.0126) (-1.526) (.965) (-3.22) (-.0076±j.538)	<u>P<sub>12e</sub></u> 3.75 (0) (-.013) (-1.52) (.965) (-3.22) (-.0076±j.538)
	<u>P<sub>21e</sub></u> 25.4 (0) (-.375±j3.58) (-.0272) (-2.7) (-.39±j2.96)	<u>P<sub>22e</sub></u> -57.4 (0) (-.354±j3.01) (-.0272 (-2.7) (-.39±j2.96)
<u>Mode</u> <u>3:</u>	<u>P<sub>11e</sub></u> -24.9 (0) (-.0126) (-1.514) (.965) (-3.22) (-.0076±j.538)	<u>P<sub>12e</sub></u> 3.24 (0) (-.013) (-1.65) (.965) (-3.22) (-.0076±j.538)
	<u>P<sub>21e</sub></u> 6.38 (0) (-.354±j2.93) (-.0272) (-2.7) (-.39±j2.96)	<u>P<sub>22e</sub></u> -22.9 (0) (-.352±j2.84) (-.0272 (-2.7) (-.39±j2.96)

\*Equivalent Plant.

TABLE C-3--Continued

<u>Mode</u> <u>4:</u>	<u>P<sub>11e</sub></u>	<u>P<sub>12e</sub></u>
	$\frac{-12.8(0) (-.0126) (-1.52)}{(.965) (-3.22) (-.0076 \pm j.538)}$	$\frac{6.24(0) (-.0125) (-1.58)}{(.965) (-3.22) (-.0076 \pm j.538)}$
	<u>P<sub>21e</sub></u>	<u>P<sub>22e</sub></u>
	$\frac{31.7(0) (-.37 \pm j3.46)}{(-.0272) (-2.7) (-.39 \pm j2.96)}$	$\frac{-31.8(0) (-.358 \pm j3.07)}{(-.0272) (-2.7) (-.39 \pm j2.96)}$
<u>Mode</u> <u>5:</u>	<u>P<sub>11e</sub></u>	<u>P<sub>12e</sub></u>
	$\frac{-12.8(0) (-.0126) (-1.52)}{(.965) (-3.22) (-.0076 \pm j.538)}$	$\frac{-.229(0) (-.012) (-3.44)}{(.965) (-3.22) (-.0076 \pm j.538)}$
	<u>P<sub>21e</sub></u>	<u>P<sub>22e</sub></u>
	$\frac{19(0) (-.382 \pm j3.77)}{(-.0272) (-2.7) (-.39 \pm j2.96)}$	$\frac{31.8(0) (-.358 \pm j3.07)}{(-.0272) (-2.7) (-.39 \pm j2.96)}$
<u>Mode</u> <u>6:</u>	<u>P<sub>11e</sub></u>	<u>P<sub>12e</sub></u>
	$\frac{-12.03(0) (-.0126) (-1.51)}{(.965) (-3.22) (-.0076 \pm j.538)}$	--
	<u>P<sub>21e</sub></u>	<u>P<sub>22e</sub></u>
	--	$\frac{-6.34(0) (-.375 \pm j3.58)}{(-.0272) (-2.7) (-.39 \pm j2.96)}$

TABLE C-4

## EQUIVALENT PLANT MATRICES FOR FLIGHT CONDITION FOUR

<u>Mode</u> <u>1:</u>	$\underline{P_{11e}^*}$ $\frac{-34.4(0) (-.03) (-1.08)}{(-.0152+j.0234) (-8.01+j6.6)}$	$\underline{P_{12e}^*}$ —
	$\underline{P_{21e}^*}$ —	$\underline{P_{22e}^*}$ $\frac{-25.8(0) (-.394+j4.47)}{(-.035) (-2.17) (-.5+j3.13)}$
<u>Mode</u> <u>2:</u>	$\underline{P_{11e}}$ $\frac{-17.9(0) (-.0302) (-1.06)}{(-.0152+j.0234) (-8.01+j6.6)}$	$\underline{P_{12e}}$ $\frac{4.11(0) (-.03) (-1.1)}{(-.0152+j.0234) (-8.01+j6.6)}$
	$\underline{P_{21e}}$ $\frac{23.3(0) (-.38+j3.85)}{(-.035) (-2.17) (-5+j3.13)}$	$\underline{P_{22e}}$ $\frac{-20(0) (-.4+j4.63)}{(-.035) (-2.17) (-.5+j3.13)}$
<u>Mode</u> <u>3:</u>	$\underline{P_{11e}}$ $\frac{-33.6(0) (-.03) (1.09)}{(-.0152+j.0234) (-8.01+j6.6)}$	$\underline{P_{12e}}$ $\frac{2.93(0) (-.035) (-.686)}{(-.0152+j.0234) (-8.01+j6.6)}$
	$\underline{P_{21e}}$ $\frac{1.77(0) (-.408+j4.92)}{(-.035) (-2.17) (-5+j3.13)}$	$\underline{P_{22e}}$ $\frac{-18.7(0) (-.39+j4.28)}{(-.035) (-2.17) (-.5+j3.13)}$

\*Equivalent Plant.

TABLE C-4--Continued

<u>Mode</u> <u>4:</u>	<u>P<sub>11e</sub></u>	<u>P<sub>12e</sub></u>
	$\frac{-17.2(0) (-.03) (-1.08)}{(-.0152+j.0234) (-8.01+j6.6)}$	$\frac{7.04(0) (-.03) (-.927)}{(-.0152+j.0234) (-.801+j6.6)}$
	<u>P<sub>21e</sub></u>	<u>P<sub>22e</sub></u>
	$\frac{25.1(0) (-.38+j3.93)}{(-.035) (-2.17) (-5+j3.13)}$	$\frac{-12.9(0) (-.394+j4.47)}{(-.035) (-2.17) (-5+j3.13)}$
<u>Mode</u> <u>5:</u>	<u>P<sub>11e</sub></u>	<u>P<sub>12e</sub></u>
	$\frac{-17.2(0) (-.03) (-1.08)}{(-.0152+j.0234) (-8.01+j6.6)}$	$\frac{1.19(0) (-.026) (-2.1)}{(-.0152+j.0234) (-.801+j6.6)}$
	<u>P<sub>21e</sub></u>	<u>P<sub>22e</sub></u>
	$\frac{21.5(0) (-.37+j3.75)}{(-.035) (-2.17) (-5+j3.13)}$	$\frac{-12.9(0) (-.39+j4.45)}{(-.035) (-2.17) (-5+j3.13)}$
<u>Mode</u> <u>6:</u>	<u>P<sub>11e</sub></u>	<u>P<sub>12e</sub></u>
	$\frac{-32.9(0) (-.03) (-1.1)}{(-.0152+j.0234) (-8.01+j6.6)}$	—
	<u>P<sub>21e</sub></u>	<u>P<sub>22e</sub></u>
	—	$\frac{-11.7(0) (-.38+j3.85)}{(-.035) (-2.17) (-5+j3.13)}$

TABLE C-5  
EQUIVALENT PLANT EQUATIONS FOR THE SINGLE INPUT-SINGLE OUTPUT SYSTEMS  
FOR FLIGHT CONDITION ONE ( $P_{-1} = [1/Q_{ij}]$ )

Mode <u>1:</u>	$\underline{Q_{11}^*}$	$\underline{Q_{12}^*}$
	$\frac{-2.18(0) (-.0175) (-.462)}{(.363) (-1.3) (-.0768+j.206)}$	--
	$\underline{Q_{21}^*}$	$\underline{Q_{22}^*}$
	--	$\frac{-5.55(0) (-.224+j1.03)}{(-.104) (-.684) (-.274+j1.91)}$
Mode <u>2:</u>	$\underline{Q_{11}}$	$\underline{Q_{12}}$
	$\frac{-.939(0) (-.0162) (-.471)}{(.363) (-1.3) (-.0768+j.206)}$	$\frac{-16.8(0) (-.203+j.847)}{(-.104) (.684) (-.274+j1.91)}$
	$\underline{Q_{21}}$	$\underline{Q_{22}}$
	$\frac{-2.2(0) (-.0162) (-.471) (-.203+j.847)}{(.363) (-1.3) (-.0768+j.207) (-.302+j1.56)}$	$\frac{-4.45(0) (-.203+j.847)}{(-.104) (-.684) (-.274+j1.91)}$

\*Equivalent Plant.

TABLE C-5--Continued

	$\underline{Q_{11}}$	$\underline{Q_{12}}$
Mode $\underline{3:}$	$\frac{-2.23(0) (-.018) (-.458)}{(.363) (-1.3) (-.0768 \pm j.206)}$	$\frac{-60.9(0) (-.458) (-.236 \pm j1.3)}{(-.104) (-.259) (-.684) (-.274 \pm j1.91)}$
	$\underline{Q_{21}}$	$\underline{Q_{22}}$
	$\frac{-13.2(0) (-.018) (-.458) (-.236 \pm j1.13)}{(.363) (-1.3) (-.078 \pm j.207) (-.205 \pm j.853)}$	$\frac{-3.34(0) (-.236 \pm j1.13)}{(-.104) (-.684) (-.274 \pm j1.91)}$
	$\underline{Q_{11}}$	$\underline{Q_{12}}$
Mode $\underline{4:}$	$\frac{-.934(0) (-.0256) (-.412)}{(.363) (-1.3) (-.0768 \pm j.206)}$	$\frac{-16.3(0) (-.226 \pm j.937)}{(-.104) (-.684) (-.274 \pm j1.91)}$
	$\underline{Q_{21}}$	$\underline{Q_{22}}$
	$\frac{-.959(0) (-.412) (-.0256) (-.226 \pm j.937)}{(.363) (-1.3) (-.0768 \pm j.207) (-.282 \pm j1.45)}$	$\frac{-2.38(0) (-.226 \pm j.937)}{(-.104) (-.684) (-.274 \pm j1.91)}$



Lower Bound--a<sub>22</sub>

$$\frac{p}{R} = \frac{6.937}{(s + 0.75)(s + 2.5)(s + 3.7)}$$

Figures of Merit:

Peak Time,  $T_p$  = Very Large

Settling Time,  $T_s$  = 5.99 sec

Peak Value,  $M_p$  = 50.0

Loop Interaction Bound--b<sub>12</sub>

$$\frac{q}{R} = \frac{0.04s (s + 1) (s + 10)}{(s + 0.75) (s + 5) (s^2 + 2s + 2)}$$

Figures of Merit:

Peak Time,  $M_p$  = 0.992 sec

Settling Time,  $T_s$  = 4.30 sec

Peak Value,  $M_p$  = 1.4 deg/sec

Loop Interaction Bound--b<sub>21</sub>

$$\frac{P}{R} = \frac{0.335s (s + 1) (s + 10)}{(s + 0.75) (s + 5) (s^2 + 2s + 2)}$$

Figures of Merit:

Peak Time,  $T_p = 0.992$  sec

Settling Time,  $T_s = 4.30$  sec

Peak Value,  $M_p = 2.344$  deg/sec

Roll Rate Models

Upper Bound--b<sub>22</sub>

$$\frac{P}{R} = \frac{2.136 (s+1.3)}{(s^2 + 2.667s + 2.776)}$$

Figures of Merit:

Peak Time,  $T_p = 1.538$  sec

Settling Time,  $T_s = 3.01$  sec

Peak Value,  $M_p = 55.0$  deg/sec

### Pitch Rate Models

Upper Bound--b<sub>11</sub>

$$\frac{g}{R} = \frac{4.167 (s + 2.667)}{(s^2 + 5.334s + 11.113)}$$

Figures of Merit:

Peak Time,  $T_p = 0.785$  sec

Settling Time,  $T_s = 1.5$  sec

Peak Value,  $M_p = 11.0$  deg/sec

Lower Bound--a<sub>11</sub>

$$\frac{g}{R} = \frac{63.75}{(s + 1.5)(s + 4.25)(s + 10)}$$

Figures of Merit:

Peak Time,  $T_p = \text{Very Large}$

Settling Time,  $T_s = 3.01$  sec

Peak Value,  $M_p = 10$  deg/sec

#### Appendix D: Pitch Rate and Roll Rate

##### Response Models

This appendix contains the transfer function models for pitch rate and roll rate. These models are synthesized with desired specifications in mind, and the models are used to develop bounds on the loop transmission. Also listed in this appendix are the figures of merit for the response models. See Chapter IV for the time and frequency domain plots of these models.

TABLE C-8--Continued

	$\underline{Q_{11}}$	$\underline{Q_{12}}$
Mode $\underline{5}$ :	$\frac{-19.2(0) (-0.03) (-1.06)}{(.152+j.0234) (-.801+j6.59)}$	$\frac{-208.2(0) (-1.16) (-.406+j4.39)}{(-.0345) (-2.17) (-2.1) (-.5+j3.13)}$
	$\underline{Q_{21}}$	$\underline{Q_{22}}$
	$\frac{-11.5(0) (-0.03) (-1.16)}{(-.0152+j.0234) (-.801+j6.59)}$	$\frac{-14.4(0) (-.406+j4.39)}{(-.0345) (-2.17) (.5+j3.13)}$
	$\underline{Q_{11}}$	$\underline{Q_{12}}$
Mode $\underline{6}$ :	$\frac{-32.9(0) (-0.03) (-1.1)}{(.152+j.0234) (-.801+j6.59)}$	--
	$\underline{Q_{21}}$	$\underline{Q_{22}}$
	--	$\frac{-11.7(0) (-.377+j3.85)}{(-.0345) (-2.17) (.5+j3.13)}$

TABLE C-8--Continued

	<u>Q<sub>11</sub></u>	<u>Q<sub>12</sub></u>
	$\frac{-33.4(0) (-.03) (-1.09)}{(.152 \pm j.0234) (-.801 \pm j6.59)}$	$\frac{-213.6(0) (-1.09) (-.388 \pm j4.28)}{(-.686) (-2.17) (-.036) (-.5 \pm j3.13)}$
Mode <u>3:</u>	<u>Q<sub>21</sub></u>	<u>Q<sub>22</sub></u>
	$\frac{-352.7(0) (-.03) (-1.09) (-.388 \pm j4.28)}{(-.0151 \pm j.0234) (-.408 \pm j4.92) (-.801 \pm j6.59)}$	$\frac{-18.6(0) (-.39 \pm j4.28)}{(-.0345) (-2.17) (.5 \pm j3.13)}$
	<u>Q<sub>11</sub></u>	<u>Q<sub>12</sub></u>
	$\frac{-3.5(0) (-.03) (-1.34)}{(.152 \pm j.0234) (-.801 \pm j6.59)}$	$\frac{-6.44(0) (-1.34) (-.62 \pm j6.08)}{(-.0345) (-.927) (-2.17) (-.5 \pm j3.13)}$
Mode <u>4:</u>	<u>Q<sub>21</sub></u>	<u>Q<sub>22</sub></u>
	$\frac{-1.81(0) (-.0286) (-1.34) (-.621 \pm j6.08)}{(-.0152 \pm j.0234) (-.38 \pm j3.93) (-.801 \pm j6.59)}$	$\frac{-2.64(0) (-.621 \pm j6.08)}{(-.0345) (-2.17) (.5 \pm j3.13)}$

TABLE C-8

EQUIVALENT PLANT EQUATIONS FOR THE SINGLE INPUT-SINGLE OUTPUT SYSTEMS  
FOR FLIGHT CONDITION FOUR ( $P_{-1} = [11Q_{ij}]$ )

<u>Mode 1:</u>	$\underline{Q_{11}^*}$	$\underline{Q_{12}^*}$
	$\frac{-34.4(0) (-.03) (-1.08)}{(.152+j.0234) (-.801+j6.59)}$	--
<u>Mode 2:</u>	$\underline{Q_{21}^*}$	$\underline{Q_{22}^*}$
	--	$\frac{-25.8(0) (-.394+j4.47)}{(-.0345) (-2.17) (.5+j3.13)}$
<u>Mode 1:</u>	$\underline{Q_{11}}$	$\underline{Q_{12}}$
	$\frac{-13.1(0) (-.0303) (-1.06)}{(.152+j.0234) (-.801+j6.59)}$	$\frac{-63.8(0) (-.405+j4.89)}{(-.0345) (-2.17) (-.5+j3.13)}$
<u>Mode 2:</u>	$\underline{Q_{21}}$	$\underline{Q_{22}}$
	$\frac{(-11.3(0) (-.0303) (-1.06) (-.405+j4.89))}{(-.0152+j.0234) (-.38+j3.85) (-.801+j6.59)}$	$\frac{-14.6(0) (-.405+j4.89)}{(-.0345) (-2.17) (.5+j3.13)}$

\*Equivalent Plant.

TABLE C-7--Continued

	$\underline{Q_{11}}$	$\underline{Q_{12}}$
Mode <u>5:</u>	$\frac{-12.98(0) (-.0126) (-1.55)}{(.965) (-3.22) (-.0076 \pm j.0538)}$	$\frac{1810.(0) (-1.55) (.354 \pm j3.08)}{(-2.7) (-3.44) (-.0272) (-.391 \pm j2.96)}$
	$\underline{Q_{21}}$	$\underline{Q_{22}}$
	$\frac{-21.8(0) (-.0126) (-1.55) (-.354 \pm j3.08)}{(.965) (-3.22) (-.0076 \pm j.0538) (-.382 \pm j3.77)}$	$\frac{-32.2(0) (-.354 \pm j3.08)}{(-.0272) (-2.7) (-.39 \pm j2.96)}$
Mode <u>6:</u>	$\underline{Q_{11}}$	$\underline{Q_{12}}$
	$\frac{-24.1(0) (-.0126) (-1.51)}{(.965) (-3.22) (-.0076 \pm j.0538)}$	—
	$\underline{Q_{21}}$	$\underline{Q_{22}}$
	—	$\frac{-12.68(0) (-.375 \pm j3.58)}{(-.0272) (-2.7) (-.39 \pm j2.96)}$



TABLE C-7--Continued

Mode		
	$\underline{Q_{11}}$	$\underline{Q_{12}}$
<u>3:</u>	$\frac{-23.97(0) (-.0126) (-1.51)}{(.965) (-3.22) (-.0076 \pm j.0538)}$	$\frac{-169.2(0) (-.352 \pm j2.84)}{(-.0272) (-2.7) (-.39 \pm j2.96)}$
	$\underline{Q_{21}}$	$\underline{Q_{22}}$
	$\frac{-85.8(0) (-.0126) (-1.61)}{(.0076) (.965) (-3.22)}$	$\frac{-22.02(0) (-.352 \pm j2.84)}{(-.0272) (-2.7) (-.39 \pm j2.96)}$
<u>4:</u>	$\underline{Q_{11}}$	$\underline{Q_{12}}$
	$\underline{Q_{21}}$	$\underline{Q_{22}}$
<u>4:</u>	$\frac{-6.62(0) (-.0127) (-1.43)}{(.965) (-3.22) (-.0076 \pm j.0538)}$	$\frac{-33.8(0) (-.364 \pm j2.64)}{(-.0272) (-2.7) (-.39 \pm j2.96)}$
	$\frac{-6.62(0) (-.013) (-1.43) (-.364 \pm j2.64)}{(.965) (-.0076 \pm j.0538) (-3.22) (-.371 \pm j3.46)}$	$\frac{-16.4(0) (-.364 \pm j2.64)}{(-.0272) (-2.7) (-.39 \pm j2.96)}$

TABLE C-7  
EQUIVALENT PLANT EQUATIONS FOR THE SINGLE INPUT-SINGLE OUTPUT SYSTEMS  
FOR FLIGHT CONDITION THREE ( $P^{-1} = [11Q_{ij}]$ )

Mode	$Q_{11}^*$	$Q_{12}^*$
$\underline{1:}$	$\frac{-25.7(0) (-0.0126) (-1.52)}{(.965) (-3.22) (-0.0076 \pm j.0538)}$	--
	$Q_{21}$	$Q_{22}^*$
	--	$\frac{-63.7(0) (-0.358 \pm j3.07)}{(-.0272) (-2.7) (-0.39 \pm j2.96)}$
Mode	$Q_{11}$	$Q_{12}$
$\underline{2:}$	$\frac{-14.98(0) (-0.0126) (-1.52)}{(.965) (-3.22) (-0.0076 \pm j.0538)}$	$\frac{285.9(0) (-0.358 \pm j3.06)}{(-.0272) (2.7) (-0.39 \pm j2.96)}$
	$Q_{21}$	$Q_{22}$
	$\frac{-33.9(0) (-0.013) (-1.5) (-0.358 \pm j3.06)}{(-.0076 \pm j.0538) (-0.375 \pm j3.58) (.965) (-3.22)}$	$\frac{-62.99(0) (-0.358 \pm j3.06)}{(-.0272) (-2.7) (-0.39 \pm j2.96)}$

\*Equivalent Plant.

TABLE C-6--Continued

<u>Mode</u> <u>5:</u>	<u>Q<sub>11</sub></u>	<u>Q<sub>12</sub></u>
	$\frac{-2.68(0) (-.0096) (-.609)}{(1.167) (-2.02) (-.0065 \pm j.078)}$	$\frac{-44.6(0) (-.208 \pm j1.63)}{(-.078) (-.326) (-.211 \pm j1.95)}$
	<u>Q<sub>21</sub></u>	<u>Q<sub>22</sub></u>
	$\frac{-6.06(0) (-.0096) (-.609) (-.208 \pm j1.63)}{(1.167) (-.0065 \pm j.078) (-.256 \pm j2.3)}$	$\frac{-9.44(0) (-.208 \pm j1.63)}{(-.078) (-.827) (-.211 \pm j1.95)}$
<u>Mode</u> <u>6:</u>	<u>Q<sub>11</sub></u>	<u>Q<sub>12</sub></u>
	$\frac{-5.86(0) (-.01) (-.55)}{(1.167) (-2.02) (-.0065 \pm j.078)}$	--
	<u>Q<sub>21</sub></u>	<u>Q<sub>22</sub></u>
	--	$\frac{-3.396(0) (-.244 \pm j2.1)}{(-.078) (-.827) (-.211 \pm j1.95)}$

TABLE C-6---Continued

<u>Mode</u> <u>3:</u>	<u>Q<sub>11</sub></u>	<u>Q<sub>12</sub></u>
	$\frac{-2.72(0) (-.0098) (-.577)}{(1.167) (-2.02) (-.0065 \pm j.078)}$	$\frac{-71.2(0) (-.218 \pm j1.61)}{(-.078) (-.827) (-.211 \pm j1.95)}$
	<u>Q<sub>21</sub></u>	<u>Q<sub>22</sub></u>
	$\frac{-7.68(0) (-.0098) (-.577) (-.218 \pm j1.61)}{(1.167) (-2.03) (-.0065 \pm j.078) (-.244 \pm j2.1)}$	$\frac{-17.5(0) (-.218 \pm j1.61)}{(-.078) (-.827) (-.211 \pm j1.95)}$
<u>Mode</u> <u>4:</u>	<u>Q<sub>11</sub></u>	<u>Q<sub>12</sub></u>
	$\frac{-2.24(0) (-.011) (-.485)}{(1.167) (-2.02) (-.0065 \pm j.078)}$	$\frac{-27.8(0) (-.485) (-.232 \pm j1.59)}{(-.078) (-.716) (-.827) (-.211 \pm j1.95)}$
	<u>Q<sub>21</sub></u>	<u>Q<sub>22</sub></u>
	$\frac{-2.6(0) (-.011) (-.485) (-.232 \pm j1.59)}{(1.167) (-2.03) (-.0065 \pm j.078) (-.238 \pm j1.99)}$	$\frac{-7.88(0) (-.232 \pm j1.59)}{(-.078) (-.827) (-.211 \pm j1.95)}$

TABLE C-6

EQUIVALENT PLANT EQUATIONS FOR THE SINGLE INPUT-SINGLE OUTPUT SYSTEMS  
FOR FLIGHT CONDITION TWO ( $P_e^{-1} = [1/Q_{ij}]$ )

<u>Mode</u> <u><math>\underline{i}</math>:</u>	<u><math>Q_{11}^*</math></u>	<u><math>Q_{12}^*</math></u>
	$\frac{-5.92(0) (-.01) (-.562)}{(1.167) (-2.02) (-.0065+j.078)}$	—
<u><math>Q_{21}^*</math></u>	<u>—</u>	$\frac{-20.8(0) (-.223+j1.7)}{(-.078) (-.827) (-.211+j1.95)}$
<u>Mode</u> <u><math>\underline{2}</math>:</u>	<u><math>Q_{11}</math></u>	<u><math>Q_{12}</math></u>
	$\frac{-5.87(0) (-.01) (-.552)}{(1.167) (-2.02) (-.0065+j.078)}$	$\frac{-577.2(0) (-.223+j1.7)}{(-.078) (-.827) (-.211+j1.95)}$
<u><math>Q_{21}</math></u>	$\frac{-28.03(0) (-.01) (-.552)}{(1.167) (-2.03) (-.0065+j.078)}$	$\frac{-10.4(0) (-.223+j1.7)}{(-.078) (-.827) (-.211+j1.95)}$

\*Equivalent Plant.

TABLE C-5--Continued

	$\underline{Q_{11}}$	$\underline{Q_{12}}$
Mode $\underline{5:}$	$\frac{-0.86(0) (-0.0071) (-0.534)}{(.363) (-1.3) (-0.0768 \pm j.206)}$	$\frac{-5.96(0) (-0.17 \pm j.743)}{(-0.104) (-0.684) (-0.274 \pm j1.91)}$
	$\underline{Q_{21}}$	$\underline{Q_{22}}$
	$\frac{-1.51(0) (-0.0071) (-0.534) (-0.17 \pm j.743)}{(.363) (-1.3) (-0.077 \pm j.207) (-0.336 \pm j1.74)}$	$\frac{-2.19(0) (-0.17 \pm j.743)}{(-0.104) (-0.684) (-0.274 \pm j1.91)}$
	$\underline{Q_{11}}$	$\underline{Q_{12}}$
Mode $\underline{6:}$	$\frac{-2.24(0) (-0.018) (-0.457)}{(.363) (-1.3) (-0.0768 \pm j.206)}$	--
	$\underline{Q_{21}}$	$\underline{Q_{22}}$
	--	$\frac{-1.07(0) (-0.302 \pm j1.56)}{(-0.104) (-0.684) (-0.274 \pm j1.91)}$

Appendix E: Templates of  $Q_{11}$  and  $Q_{22}$

This appendix contains templates of  $Q_{11}$  and  $Q_{22}$   
at various frequencies.

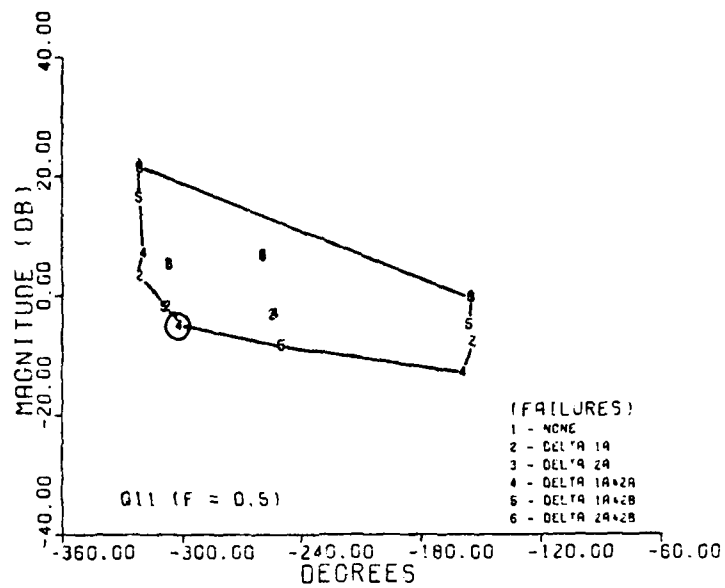


Fig. E-1(a). Template of  $Q_{11}$  at  $\omega = 0.5$  Rad/Sec

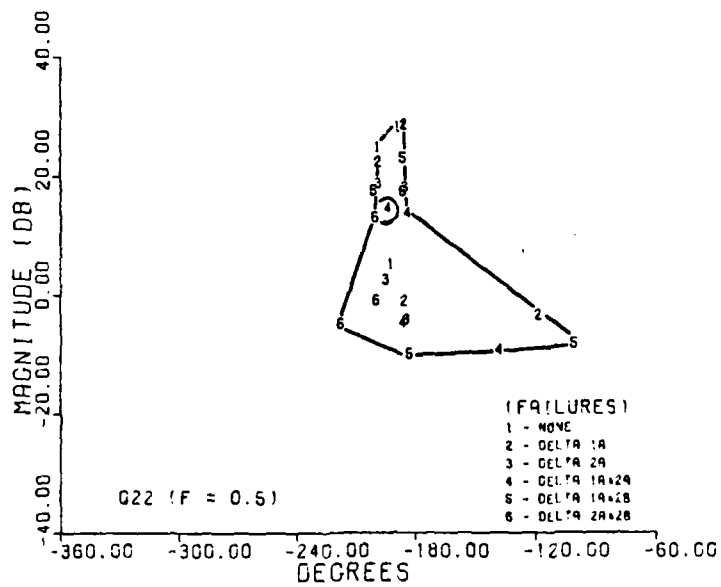


Fig. E-1(b). Template of  $Q_{22}$  at  $\omega = 0.5$  Rad/Sec



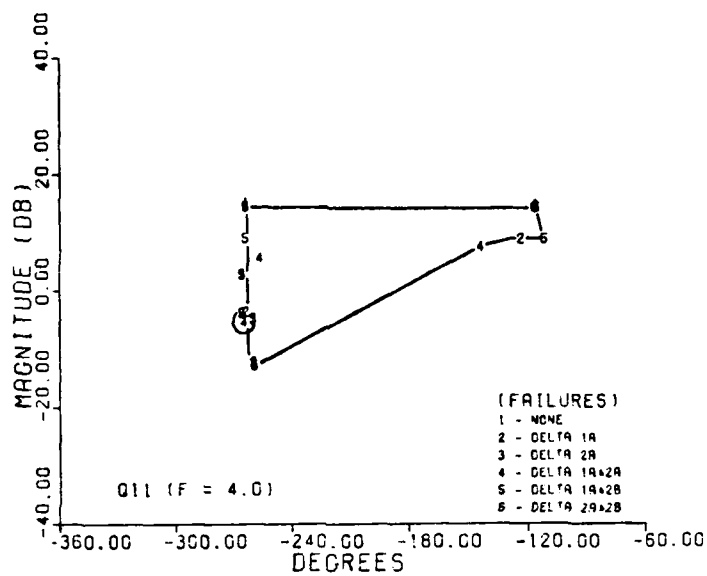


Fig. E-2(a). Template of  $Q_{11}$  at  $\omega = 4.0$  Rad/Sec

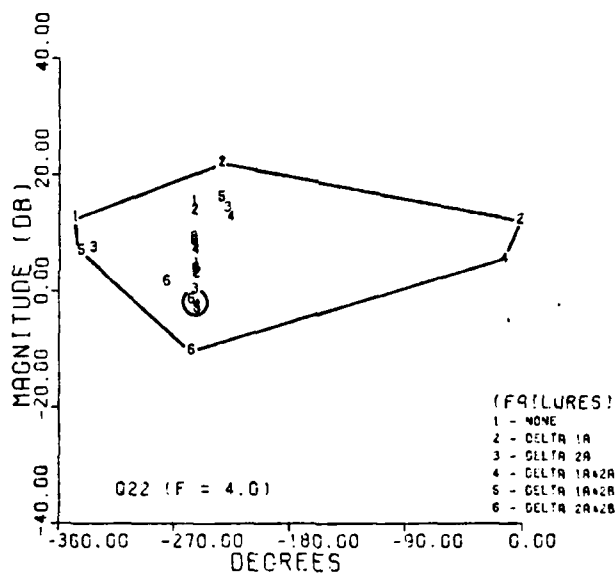


Fig. E-2(b). Template of  $Q_{22}$  at  $\omega = 4.0$  Rad/Sec

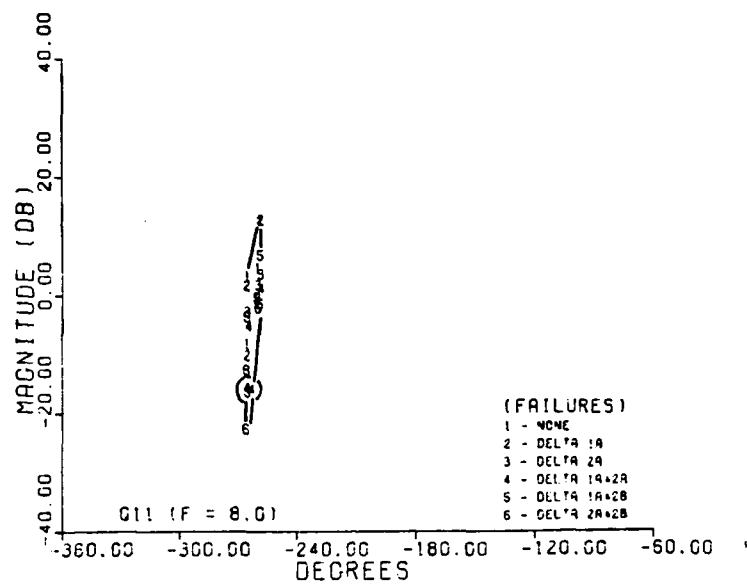


Fig. E-3(a). Template of  $Q_{11}$  at  $\omega = 8$  Rad/Sec

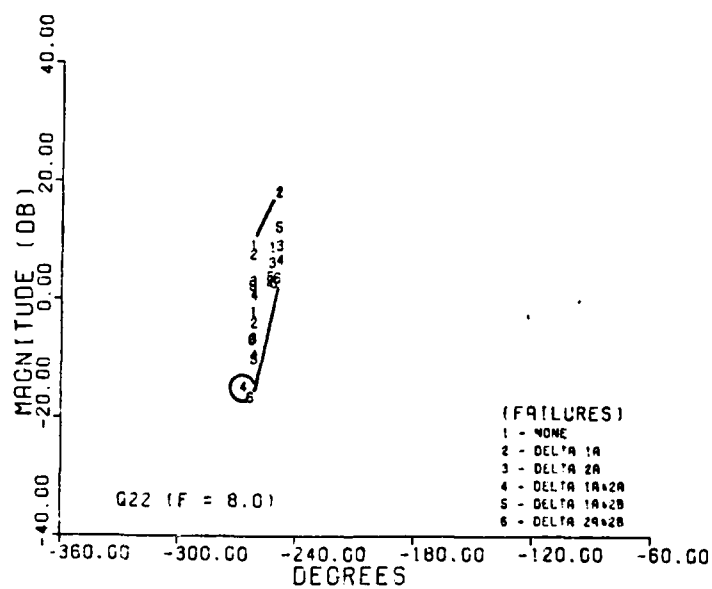


Fig. E-3(b). Template of  $Q_{22}$  at  $\omega = 8$  Rad/Sec

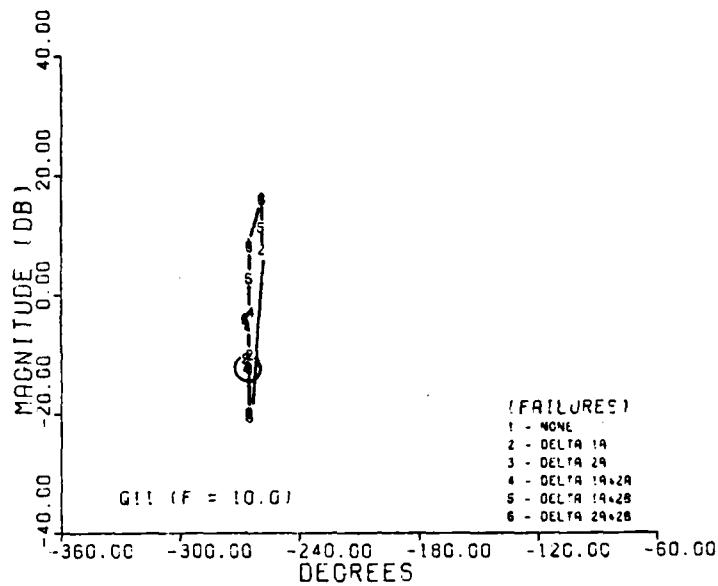


Fig. E-4(a). Template of  $Q_{11}$  at  $\omega = 10$  Rad/Sec

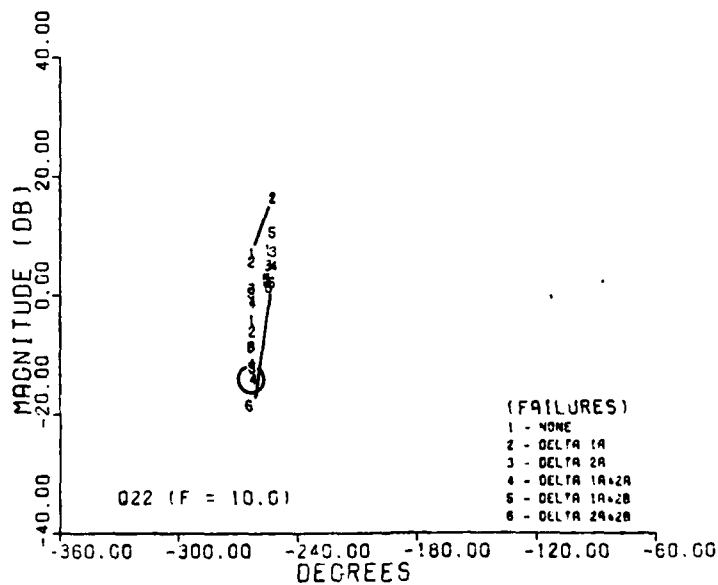


Fig. E-4(b). Template of  $Q_{22}$  at  $\omega = 10$  Rad/Sec

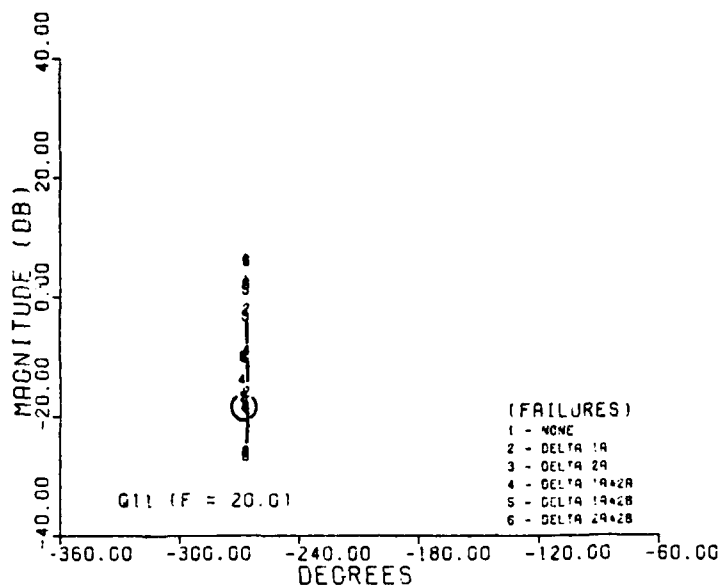


Fig. E-5(a). Template of  $Q_{11}$  at  $\omega = 20$  Rad/Sec

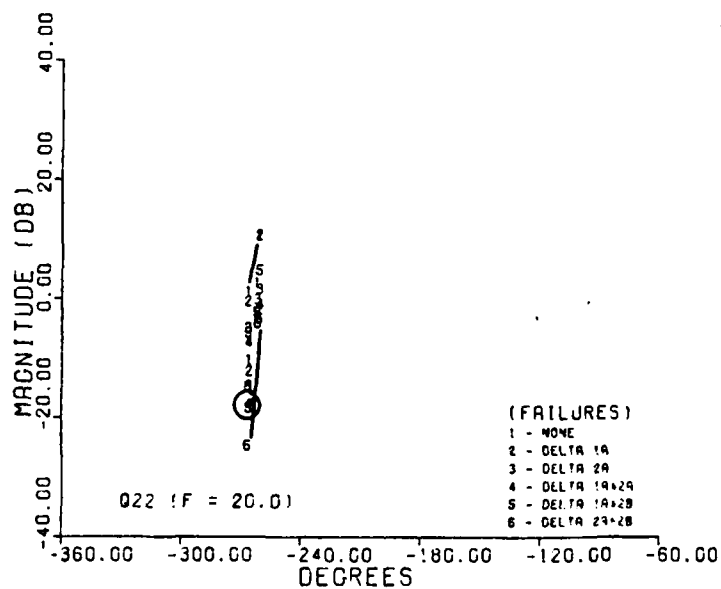


Fig. E-5(b). Template of  $Q_{22}$  at  $\omega = 20$  Rad/Sec

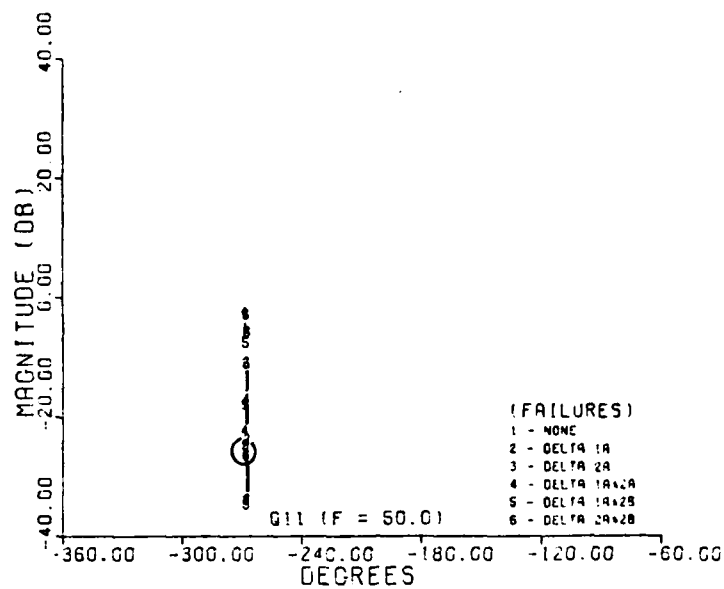


Fig. E-6(a). Template of  $Q_{11}$  at  $\omega = 50$  Rad/Sec

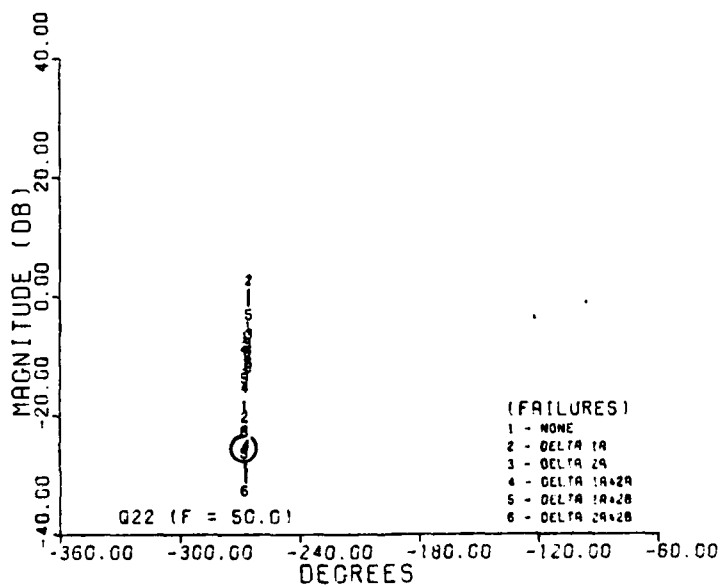


Fig. E-6(b). Template of  $Q_{22}$  at  $\omega = 50$  Rad/Sec

## Appendix F: Simulation Programs

The computer programs listed in this appendix are the programs used to conduct the simulation of the system represented by Figure 2-1. The simulation is structured around a main program and three subroutines. The main program is called Zsimu; Zsimu calls subroutines Derv, Zdata, and Degol. Poles and zeros of the basic plant equations are listed in Zdata.

Data for the four basic plant equations ( $P_{ij}$ ) at each flight condition are listed in Zdata. These data are passed as poles and zeros to Zsimu. Derivatives of the output variables are passed from Derv to Zsimu.

Subroutine Derv uses the state-variable format to write basic plant, prefilter, and compensator equations. Desired outputs are derived from the state equations based upon the structure of the system being simulated. For this simulation, a maximum of 30 state-variables can be defined in Derv. This limitation is due to the subroutine Degol. Position saturation for the elevators and flaperons is also included in this subroutine. Derivatives of the output variables are passed to the main program for integration.

The main program Zsimu takes the outputs of Derv and integrates these using the subroutine Degol. Degol is an integration routine for linear, ordinary differential equations similar to the program ODE. The "case" block of Zsimu allows inputs (IR), flight conditions (IC), or CSC modes (IM) to be selected as desired. The "integration" block allows the integration time and the number of points in the integration to be specified. For this simulation the outputs of Zsimu are pitch rate, roll rate, and the four surface deflections. These outputs are identified by the variables YY1 to YY6, respectively. Integration time is denoted by the variable XX. If other outputs are desired, they are defined in Derv. A step or ramp command input may be specified in the "Degol" block.

# Zsimu

PROGRAM ZSIMU(INPUT,OUTPUT,DATALP,DATAOUT,PLOT,TAPE9,  
+TAPE6=OUTPUT,TAPE7=DATALP,TAPE8=DATAOUT)

```
C
C
C  UPDATE% AUG. 15, 1984
C  ORIGINAL% AUG.12, 1984
C
C*****      DIMENSION      *****
C
C
C-----  DEGOL  -----
      REAL Y(30)
      EXTERNAL DERV
      DIMENSION XX(500),YY(500),YY1(500),YY2(500),YY3(500),
      DIMENSION YY4(500),YY5(500),YY6(500)
C
C-----  PLANT  -----
C
      DIMENSION VP11K(4),VP11Z1(4),VP11Z2(4),VP11Z3(4)
C
      DIMENSION VP12K(4),VP12Z1(4),VP12Z2(4),VP12Z3(4)
C
      COMPLEX VP11P1(4),VP11P2(4),VP11P3(4),VP11P4(4)
C
      DIMENSION VP21K(4),VP21Z1(4)
      COMPLEX VP21Z2(4),VP21Z3(4)
C
      DIMENSION VP22K(4),VP21()
      COMPLEX VP22Z2(4),VP22Z3(4)
C
      DIMENSION VP22P1(4),VP22P2(4)
      COMPLEX VP22P3(4),VP22P4(4)
C
C*****      COMMON      *****
C
C-----  DERV  -----
C
      COMMON/P11Z/ P11K,P11Z1,P11Z2,P11Z3
      COMMON/P11P/ P11P1,P11P2,P11P3,P11P4
C
      COMMON/P12Z/ P12K,P12Z1,P12Z2,P12Z3
C
      COMMON/P21Z/ P21K,P21Z1,P21Z2,P21Z3
```



```

C      COMMON /P22Z/ P22K,P22Z1,P22Z2,P22Z3
      COMMON /P22P/ P22P1,P22P2,P22P3,P22P4
C
C      COMMON /F/ F1K,F1P,F2K,F2P
C
      COMMON /G1Z/G1K,G1Z1,G1Z2,G1Z3,G1Z4
      COMMON /G1P/G1P1,G1P2,G1P3,G1P4
C
      COMMON /G2Z/G2K,G2Z1,G2Z2,G2Z3,G2Z4
      COMMON /G2P/G2P1,G2P2,G2P3,G2P4
C
      COMMON /R/ R1,R2
      COMMON /U/ U1,U2
      COMMON /C/ C1,C2
      COMMON /D/ D1A,D1B,D2A,D2B
      COMMON /IS/ IS1A,IS1B,IS2A,IS2B
C
C----- MONITOR -----
      COMMON /M/ F1,F2,G1,G2,E1,E2,D11,D12,D21,D22,
      COMMON /M/ C11,C12,C21,C22
C----- ZDATA -----
C
      COMMON /VP11Z/VP11K,VP11Z1,VP11Z2,VP11Z3
      COMMON /VP11P/VP11P1,VP11P2,VP11P3,VP11P4
C
      COMMON /VP12Z/VP12K,VP12Z1,VP12Z2,VP12Z3
C
      COMMON /VP21Z/VP21K,VP21Z1,VP21Z2,VP21Z3
C
      COMMON /VP22Z/VP22K,VP22Z1,VP22Z2,VP22Z3
      COMMON /BP22P/VP22P1,VP22P2,VP22P3,VP22P4
C
C----- CASE -----
C
      DIMENSION IRV(30),ICV(30),IMV(30)
C
      DATA NC/1/
      DATA IRV/1,1,1,1,1,1,2,23*0/
      DATA ICV/2,2,4,2,4,2,2,23*0/
      DATA IMV/4,2,3,4,5,6,7,23*0/
C
C***** MAIN PROGRAM *****
C
C----- P(S) -----
C
      CALL ZDATA
C
C----- F1(S) -----

```

```

      F1K = 2.
      F1P = 2.
C
C----- F2(S) -----
      F2K = 2.
C
      F2P = 2.
C
C----- G1(S) -----
      G1K = -660000.
C
      G1Z1 = 4.
      G1Z2 = 100.
      G1Z3 = .013
      G1Z4 = .006
C      G1Z3 = .013 + .006
C      G1Z4 = .013*.006
C
      G1P1 = .0223
      G1P2 = 25.
C      G1P3 = 630.
C      G1P4 = 630.
      G1P3 = 60 + 630.
      G1P4 = 630.*630.
C
C----- G2(S) -----
      G2K = -36800.
      G2Z1 = 3.
      G2Z2 = 900.
      G2Z3 = 2.
      G2Z4 = 4.
C      G2Z3 = 1. + 10.
C      G2Z4 = 10.
C
      G2P1 = .25
      G2P2 = 50.
C      G2P3 = 158.
C      G2P4 = 3750.
      G2P3 = 158. + 3750.
      G2P4 = 158.*3750.
C
C
C----- INPUT R(I) -----
      R1 = 10.
      R2 = 50.
C
C----- SCALE U(I,J) -----
      U12 = .25
      U21 = .25
C
C

```

```

C***** DO CASES *****
C
C      DO 10 III = 1,NC
C
C          IR = IRV(III)
C          IC = ICV(III)
C          IM = IMV(III)
C          R1 = 0.
C          R2 = 0.
C          FIR.EQ. 1) R1 = 10.
C          IF(IR .EQ. 2) R2 = 50.
C
C          IS1A = 1
C          IS1B = 1
C          IS2A = 1
C          IS2B = 1
C          IF(IM .EQ. 2) IS1A = 0
C          IF(IM .EQ. 3) IS2A = 0
C          IF(IM .EQ. 4) IS1A = 0
C          IF(IM .EQ. 4) IS2A = 0
C          IF(IM .EQ. 5) IS1A = 0
C          IF(IM .EQ. 5) IS2B = 0
C          IF(IM .EQ. 6) IS2A = 0
C          IF(IM .EQ. 6) IS2B = 0
C          IF(IM .EQ. 7) IS1A = 0
C          IF(IM .EQ. 7) IS1B = 0
C
C----- P11(S) -----
C          P11K = VP11K(IC)*.5
C          P11Z1 = -VP11Z1(IC)
C          P11Z2 = VP11Z2(IC)
C          P11Z3 = VP11Z3(IC)
C          P11Z2 = -VP11Z2(IC) - VP11Z3(IC)
C          P11Z3 = VP11Z2(IC) 1Z3(IC)
C
C          P11P1 = -2.*REAL(VP11P1(IC))
C          P11P2 = CABS(VP11P1(IC))*2
C          P11P1 = -REAL(VP11P1(IC)+VP11P2(IC))
C          P11P2 = REAL(VP11P1(IC)*VP11P2(IC))
C          P11P3 = -2.*REAL(VP11P3(IC))
C          P11P4 = CABS(VP11P3(IC))*2
C----- P12(S) -----
C          P12K = VP12K(IC)*.5
C          P12Z1 = -VP12Z1(IC)
C          P12Z2 = VP12Z2(IC)
C          P12Z3 = VP12Z3(IC)
C          P12Z2 = -VP12Z2(IC) - VP12Z3(IC)
C          P12Z3 = VP12Z2(IC)*VP12Z3(IC)
C
C----- P21(S) -----
C          P21K = VP21K(IC)*.5

```

```

      P21Z1 = -VP21Z1(IC)
      P21Z2 = -2.*REAL(VP21Z2(IC))
      P21Z3 = CABS(VP21Z2(IC))*2
C
C----- P22(S) -----
      P22K =VP22K(IC)*.5
      P22Z1 = -VP22Z1(IC)
      P22Z2 = -2.*REAL(VP22Z2(IC))
      P22Z3 = CABS(VP22Z2(IC))*2
C
C      P22P1 = VP22P1(IC)
C      P22P2 = VP22P2(IC)
C      P22P1 = -VP22P1(IC) - VP22P2(IC)
      P22P2 = VP22P1(IC)*VP22P2(IC)
      P22P3 = -2.*REAL(VP22P3(IC))
      P22P4 = CABS(VP22P3(IC))*2
C
C
C***** INTEGRATION *****
C
      NE = 28
      AE = 1.E-5
      RE = 1.E-6
      MAX = 1000
      H = 1.E-4
      N = 60000
      NP = 300
      T0 = 0.
      T1 = H
C
C----- INITIAL CONDITION -----
      C1 = 0.
      C2 = 0.
      D1A = 0.
      D1B = 0.
      D2A = 0.
      Z = 0.
C
      DO 30 I = 1,NE
30      Y(I) = 0.
C
C----- ITERATION -----
      II = 0
      IIXY = 1
      KKXY = N/NP
      IF(KKXY .LT. 1) KKXY = 1
C
      DO 40 I = 1,N
C

```

AD-A151 771

FLIGHT CONTROL SYSTEM RECONFIGURATION DESIGN USING  
QUANTITATIVE FEEDBACK THEORY(U) AIR FORCE INST OF TECH  
WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI.. P B ARNOLD  
DEC 84 AFIT/GE/ENG/84D-15 F/G 1/4

3/3

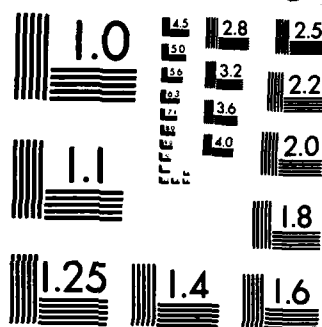
UNCLASSIFIED

NL

END

FILED

DTIC



MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS-1963-A

```

C----- YY(II) -----
          IF(I .EQ. IIXY) THEN
            II = II + 1
C          WRITE(8,122) II,G1,G2,D11,D12,D21,D22
C          WRITE(8,122) II,C1,C2,C11,C12,C21,C22
          PRINT 122,II,T0,C1,C2,D1A,D1B,D2A,D2B
          XX(II) = T0
          YY1(II) = C1
          YY2(II) = C2
          YY3(II) = D1A
          YY4(II) = D1B
          YY5(II) = D2A
          YY6(II) = D2B
          IIXY = IIXY + KKXY
          END IF
C
C----- CALL DEGOL -----
C          PRINT 122,I,T0,C1,C2,D1A,D1B,D2A,D2B
          IK = 1
          CALL DEGOL(DERV,NE,Y,T0,T1,RE,AE,IK,MAX)
          IF(IK .NE. 2) THEN
            PRINT *,IK,T0,T1
            STOP
          END IF
          T1 =T1 + H
C
C          IF(T1 .GT. 5.) R1 = 0.
C          IF(T1 .GT. 5.) R2 = 0.
C          IF(IR .EQ. 1 .AND. T1 .LE. .5) R1 = 20.*T1
          IF(IR .EQ. 1 .AND. T1 .GT. .5) R1 = 10.
          IF(IR .EQ. 1 .AND. T1 .GE. 5.) R1 = 0.
C
          IF(IR .EQ. 2 .AND. T1 .LE. 1.) R2 = 50.*T1
          IF(IR .EQ. 2 .AND. T1 .GT. 1.) R2 = 50.
          IF(IR .EQ. 2 .AND. T1 .GE. 5.) R2 = 0.
C
40    CONTINUE
C
C***** PLOT *****
          NY = II
          PRINT *,NXY
C
          WRITE(8,121)NXY
C          WRITE(7,121)NXY
          DO 50 I = 1,NXY
            WRITE(8,122) I,XX(I),YY1(I),YY2(I),YY3(I),
              YY4(I),YY5(I),YY6(I)
            WRITE(7,122) I,XX(I),YY1(I),YY2(I),YY3(I),
              YY4(I),YY5(I),YY6(I)
50    CONTINUE
C
          CALL LPL(NXY, XX, YY1,IM)

```

```

C
C----- END CASES LOOP -----
10  CONTINUE
C
C***** PRINT OUT *****
C
      CALL LPRINT(1)
      CALL LPL1(NXY,XX,YY2,1)
      CALL LPRINT(2)
C
C
C***** FORMAT *****
121  FORMAT(1X,I5)
122  FORMAT(1X,I5,7(1X,E9.3))
C
      RETURN
      END
C

```



# Degol

```
C      SUBROUTINE DEGOL
C
      SUBROUTINE DEGOL (F,NEQN,Y,T,TOUT,RELERR,ABSERR,IFLAG,
        ITMAX)
      INTEGER K
      REAL HOLD
      LOGICAL START,CRASH,STIFF
      DIMENSION Y (NEQN),PSI (12)
      DIMENSION YY (30),WT (30),PHI (30,16),P (30),YP (30),
        YPOUT (30)
      EXTERNAL F
      DATA FOURU/.44E-15/
      MAXNUM=MAX0 (500,ITMAX)
      IF (NEQN .LT. 1 .OR. NEQN .GT. 30) GO TO 10
      IF (T .EQ. TOUT) GO TO 10
      IF (RELERR .LT. 0.0 .OR. ABSERR .LT. 0.0) GO TO 10
      EPS = AMAX1 (RELERR,ABSERR)
      IF (EPS .LE. 0.0) GO TO 10
      IF (IFLAG .EQ. 0) GO TO 10
      ISN = ISIGN (1,IFLAG)
      IFLAG = IABS (IFLAG)
      IF (IFLAG .EQ. 1) GO TO 20
      IF (IFLAG .GE. 2 .AND. IFLAG .LE. 5) GO TO 20
10     IFLAG = 6
      RETURN
20     DEL = TOUT - T
      ABSDEL = ABS (DEL)
      TEND = T + 10.0*DEL
      IF (ISN .LT. 0) TEND = TOUT
      NOSTEP = 0
      KLE4 = 0
      STIFF = .FALSE.
      RELEPS = RELERR/EPS
      ABSEPS = ABSERR/EPS
      IF (IFLAG .EQ. 1) GO TO 30
      IF (ISNOLD .LT. 0) GO TO 30
      IF (DELSGN*DEL .GT. 0.0) GO TO 50
C
C      ON START AND RESTART ALSO SET WORK VARIABLES X AND YY (*),
C      STORE THE DIRECTION OF INTEGRATION AND INITIALIZE THE
C      STEP SIZE
C
30     START = .TRUE.
      X = T
      DO 40 L = 1,NEQN
```

```

40      YY(L) = Y(L)
      DELSGN = SIGN(1.0,DEL)
      H = SIGN(AMAX1(ABS(TOUT-X),FOURU*ABS(X)),TOUT-X)
C
C      IF ALREADY PAST OUTPUT POINT, INTERPOLATE AND RETURN
C
50      IF(ABS(X-T) .LT. ABSDEL) GO TO 60
      CALL INTRP(X,YY,TOUT,Y,YPOUT,NEQN,KOLD,PHI,PSI)
      IFLAG = 2
      T = TOUT
      TOLD = T
      ISNOLD = ISN
      RETURN
C
C      IF CANNOT GO PAST OUTPUT POINT AND SUFFICIENTLY CLOSE,
C      EXTRAPOLATE AND RETURN
C
60      IF(ISN .GT. 0 .OR. ABS(TOUT-X) .GE. FOURU*ABS(X))
          GO TO 80
      H = TOUT - X
      CALL F(X,YY,YP)
      DO 70 L = 1,NEQN
70      Y(L) = YY(L) + H*YP(L)
      IFLAG = 2
      T = TOUT
      TOLD = T
      ISNOLD = ISN
      RETURN
C
C      TEST FOR TOO MUCH WORK
C
80      IF(NOSTEP .LT. MAXNUM) GO TO 100
      IFLAG = ISN*4
      IF(STIFF) IFLAG = ISN*5
      DO 90 L = 1,NEQN
90      Y(L) = YY(L)
      T = X
      TOLD = T
      ISNOLD = 1
      RETURN
C
C      LIMIT STEP SIZE, SETWEIGHT VECTOR AND A STEP
C
100     H = SIGN(AMIN1(ABS(H),ABS(TEND-X)),H)
      DO 110 L = 1,NEQN
110     WT(L) = RELEPS*ABS(YY(L)) + ABSEPS
      CALL STEP(X,YY,F,NEQN,H,EPS,WT,START,
+      HOLD,K,KOLD,CRASH,PHI,P,YP,PSI)
C
C      TEST FOR TOLERANCE S TOO SMALL
      IF(.NOT. CRASH) GO TO 130

```

```

        IFLAG = ISN*3
        RELERR = EPS*RELEPS
        ABSERR = EPS*ABSEPS
        DO 120 L = 1,NEQN
120      Y(L) = YY(L)
        T = X
        TOLD = T
        ISNOLD = 1
        RETURN
C
C      AUGMENT COUNTER ON WORK AND TEST FOR STIFFNESS
C
130    NOSTEP = NOSTEP + 1
        KLE4 = KLE4 + 1
        IF(KOLD .GT. 4) KLE4 = 0
        IF(KLE4 .GE. 50) STIFF = .TRUE.
        GO TO 50
        END
C*****
C      SUBROUTINE STEP
C*****
        SUBROUTINE STEP(X,Y,F,NEQN,H,EPS,WT,START,
+      HOLD,K,KOLD,CRASH,PHI,P,YP,PSI)
        LOGICAL START,CRASH,PHASE1,NORND
        DIMENSION Y(NEQN),WT(NEQN),PHI(NEQN,16),P(NEQN),
+      YP(NEQN),PSI(12)
        DIMENSION ALPHA(12),BETA(12),SIG(13),W(12),V(12),G(13),
+      GSTR(13),TWO(13)
        EXTERNAL F
        DATA TWOU/.22E-15/
        DATA FOURU/.44E-15/
        DATA TWO/2.0,4.0,8.0,16.0,32.0,64.0,128.0,256.0,512.0,
+      1024.0,2048.0,4096.0,8192.0/
        DATA GSTR/0.500,0.0833,0.0417,0.0264,0.0188,0.0143,
+      0.0114,0.00936,0.00789,0.00679,0.00592,0.00524,
+      0.00468/
        DATA G(1),G(2)/1.0,0.5/,SIG(1)/1.0/
        CRASH = .TRUE.
        IF(ABS(H) .GE. FOURU*ABS(X)) GO TO 5
        H = SIGN(FOURU*ABS(X),H)
        RETURN
5      P5EPS = 0.5*EPS
C
C      IF ERROR TOLERANCE IS TOO SMALL, INCREASE IT TO AN
C      ACCEPTABLE VALUE
C
        ROUND = 0.0
        DO 10 L = 1,NEQN
10      ROUND = ROUND + (Y(L)/WT(L))**2
        ROUND = TWOU*SQRT(ROUND)

```

```

        IF(P5EPS .GE. ROUND) GO TO 15
        EPS = 2.0*ROUND*(1.0 + FOURU)
        RETURN
15    CRASH = .FALSE.
        IF(.NOT.START) GO TO 99
C
C    INITIALIZE.    COMPUTE APPROPRIATE STEP SIZE FOR FIRST
C    STEP
C        CALL F(X,Y,YP)
        SUM = 0.0
        DO 20 L = 1,NEQN
            PHI(L,1) = YP(L)
            PHI(L,2) = 0.0
20    SUM = SUM + (YP(L)/WT(L))**2
        SUM = SQRT(SUM)
        ABSH = ABS(H)
        IF(EPS.LT. 16.0*SUM*H*H) ABSH = 0.25*SQRT(EPS/SUM)
        H = SIGN(AMAX1(ABSH,FOURU*ABS(X)),H)
        HOLD = 0.0
        K = 1
        KOLD = 0
        START = .FALSE.
        PHASE1 = .TRUE.
        NORND = .TRUE.
        IF(P5SEPS
.GT. 100.0*ROUND) GO TO 99
        NORND = .FALSE.
        DO 25L = 1,NEQN
25    PHI(L,15) = 0.0
99    IFAIL = 0
C        ***    END BLOCK 0    ***
C        ***    BEGIN BLOCK 1    ***
C    COMPUTE COEFFICIENTS OF FORMULAS FOR THIS STEP.  AVOID
C    COMPUTING THOSE QUANTITIES NOT CHANGED WHEN STEP SIZE IS
C    NOT CHANGED.
C        ***
C
100   KP1 = K+1
        KP2 = K+2
        KM1 = K-1
        KM2 = K-2
C
C    NS IS THE NUMBER OF STEPS TAKEN WITH SIZE H, INCLUDING
C    THE CURRENT ONE.  WHEN K.LT.NS,  NO COEFFICIENTS CHANGE
C
        IF(N .NE. HOLD) NS = 0
        NS = MIN0(NS+1,KOLD+1)
        NSP1 = NS+1
        IF(K .LT. NS) GO TO 199
C

```

```

C   COMPUTE THOSE COMPONENTS OF ALPHA(*),BETA(*)PSI(*),
C   SIG(*) WHICH ARE CHANGED
      BETA(NS) = 1.0
      REALNS = NS
      ALPHA(NS) = 1.0/REALNS
      TEMP1 = H*REALNS
      SIG(NSP1) = 1.0
      IF(K .LT. NSP1) GO TO 110
      DO 105 I = NSP1,K
        IM1 = I-1
        TEMP2 = PSI(IM1)
        PSI(IM1) = TEMP1
        BETA(I) = BETA(IM1)*PSI(IM1)/TEMP2
        TEMP1 = TEMP2 + H
        ALPHA(I) = H/TEMP1
        REALI = I
105     SIG(I+1) = REALI*ALPHA(I)*SIG(I)
110    PSI(K) = TEMP1
C
C   COMPUTE COEFFICIENTS G(*)
C
C   INITIALIZE V(*) AND W(*), G(2) IS SET DATA STATEMENT
C   IF(NS .GT. 1) GO TO 120
      DO 115 IQ = 1,K
        TEMP3 = IQ*(IQ+1)
        V(IQ) = 1.0/TEMP3
115     W(IQ) = V(IQ)
      GO TO 140
C
C   IF ORDER WAS RAISED, UPDATE DIAGONAL PART OF V(*)
C
C   120 IF(K .LE. KOLD) GO TO 130
      .TEMP4 = K*KP1
      V(K) = 1.0/TEMP4
      NSM2 = NS-2
      IF(NSM2 .LT. 1) GO TO 130
      DO 125 J = 1,NSM2
        I = K-J
125     V(I) = V(I) - ALPHA(J+1)*V(I+1)
C
C
C   UPDATE V(*) AND SET W(*)
C
C   130 LIMIT1 = KP1 - NS
      TEMP5 = ALPHA(NS)
      DO 135 IQ = 1,LIMIT1
        V(IQ) = V(IQ) - TEMP5*V(IQ+1)
135     W(IQ) = V(IQ)
      G(NSP1) = W(1)
C

```

```

C   COMPUTE THE G(*) IN THE WORK VECTOR W(*)
C
140  NSP2 = NS +2
      IF(KP1 .LT. NSP2) GO TO 199
      DO 150 I = NSP2,KP1
          LIMIT2 = KP2 -I
          TEMP 6 = ALPHA(I-1)
      DO 145 IQ = 1,LIMIT2
145   W(IQ) = W(IQ) - TEMP6*W(IQ+1)
150   G(I) = W(1)
199  CONTINUE
C      ***      END BLOCK 1      ***
C
C      ***      BEGIN BLOCK 2      ***
C   PREDICT A SOLUTION P(*), EVALUATE DERIVATIVES USING
C   PREDICTED SOLUTION, ESTIMATE LOCAL ERROR AT ORDER K
C   AND ERRORS AT ORDERS K, K-1, K-1 AS IF CONSTANT STEP
C   SIZE WERE USED.
C
C   CHANGE PHI TO PHI STAR
C
      IF(K .LT. NSP1) GO TO 215
      DO 210 I = NSP1,K
          TEMP1 = BETA(I)
          DO 205 L = 1,NEQN
205   PHI(L,I) = TEMP1*PHI(L,I)
210   CONTINUE
C
C   PREDICT SOLUTION AND DIFFERENCES
C
215  DO 220 L = 1,NEQN
      PHI(L,KP2) = PHI(L,KP1)
      PHI(L,KP1) = 0.0
220  P(L) = 0.0
      DO 230 J = 1,K
          I = KP1 - J
          IP1 = I+1
          TEMP2 = G(I)
          DO 225 L = 1,NEQN
225   P(L) = P(L) + TEMP2*PHI(L,I)
          PHI(L,I) = PHI(L,I) + PHI(L,IP1)
230  CONTINUE
      IF(NORND) GO TO 240
      DO 235 L = 1,NEQN
          TAU = H*P(L) - PHI(L,15)
          P(L) = Y(L) + TAU
235   PHI(L,16) = (P(L) - Y(L)) - TAU
          GO TO 250
240  DO 245 L = 1,NEQN

```

```

245      P(L) = Y(L) + H*P(L)
250      XOLD = X
          X = X + H
          ABSH = ABS(H)
          CALL F(X,P,YP)
C
C      ESTIMATE ERRORS AT ORDERS K, K-1, K-2
C
          ERKM2 = 0.0
          ERKM1 = 0.0
          ERK = 0.0
          DO 265 L = 1,NEQN
              TEMP3 = 1.0/WT(L)
              TEMP4 = YP(L) - PHI(L,1)
              IF(KM2)265,260,255
255      ERKM2 = ERKM2 + ((PHI(L,KM1)+TEMP4)*TEMP3)**2
260      ERKM1 = ERKM1 + ((PHI(L,K)+TEMP4)*TEMP3)**2
265      ERK = ERK + (TEMP4*TEMP3)**2
              IF(KM2)280,275,270
270      ERKM2 = ABSH*SIG(KM1)*GSTR(KM2)*SQRT(ERKM2)
275      ERKM1 = ABSH*SIG(K)*GSTR(KM1)*SQRT(ERKM1)
280      TEMP5 = ABSH*SQRT(ERK)
          ERR = TEMP5*(G(K) - G(KP1))
          ERK = TEMP5*SIG(KP1)*GSTR(K)
          KNEW = K
C
C      TEST IF ORDER SHOULD BE LOWERED
C
          IF(KM2)299,290,285
285      IF(AMAX1(ERKM1,ERKM2) .LE. ERK) KNEW = KM1
          GO TO 299
290      IF(ERKM1 .LE. 0.5*ERK) KNEW = KM1
C
C      TEST IF STEP SUCCESSFUL
C
299      IF(ERR .LE. EPS) GO TO 400
C
C      ***      END BLOCK 2      ***
C
C      ***      BEGIN BLOCK 3      ***
C      THE STEP IS UNSUCCESSFUL.  RESTORE X, PHI(*,*), PSI(*).
C      IF THIRD CONSECUTIVE FAILURE, SET ORDER TO ONE.  IF
C      STEP FAILS MORE THAN THREE TIMES, CONSIDER AN OPTIMAL
C      STEP SIZE.  DOUBLE ERROR TOLERANCE AND RETURN IF
C      ESTIMATED STEP SIZE IS TOO SMALL FOR MACHINE PRECISION.
C      ***
C
C      RESTORE X, PHI(*,*) AND PSI(*)
C

```

```

    PHASE1 = .FALSE.
    X = XOLD
    DO 310 I = 1,K
        TEMP1 = 1.0/BETA(I)
        IPL = I+1
        DO 305 L = 1,NEQN
305     PHI(L,I) = TEMP1*(PHI(L,I) - PHI(L,IPL))
310     CONTINUE
        IF(K .LT. 2) GO TO 320
        DO 315 I = 2,K
315     PSI(I-1) = PSI(I) - H

C   ON THIRD FAILURE, SET ORDER TO ONE.   THEREAFTER, USE
C   OPTIMAL STEP SIZE
C   IFAIL = IFAIL + 1
    TEMP2 = 0.5
    IF(IFAIL - 3) 335,330,325
325  IF(P5EPS .LT. 0.25*ERK) TEMP2 = SQRT(P5EPS/ERK)
330  KNEW = 1
335  H = TEMP2*H
    K = KNEW
    IF(ABS(H) .GE. FOURU*ABS(X)) GO TO 340
    CRASH = .TRUE.
    H = SIGN(FOURU*ABS(X),H)
    EPS = EPS + EPS
    RETURN
340  GO TO 100
C   ***   END BLOCK 3   ***
C
C   ***   BEGIN BLOCK 4   ***
C   THE STEP IS SUCCESSFUL.  CORRECT THE PREDICTED SOLUTION,
C   EVALUATE THE DERIVATIVES USING THE CORRECTED SOLUTION
C   AND UPDATE THE DIFFERENCES.  DETERMINE BEST AND SIZE FOR
C   NEXT STEP.
C
400  KOLD = K
    HOLD = H
C
C   CORRECT AND EVALUATE
C
    TEMP1 = H*G(KP1)
    IF(NORND) GO TO 410
    DO 405 L = 1,NEQN
        RHO = TEMP1*(YP(L) - PHI(L,1)) - PHI(L,16)
        Y(L) = P(L) + RHO
405  PHI(L,15) = (Y(L) - P(L)) - RHO
        GO TO 420
    DO 415 L = 1,NEQN
415  Y(L) = P(L) + TEMP1*(YP(L) - PHI(L,1))
420  CALL F(X,Y,YP)
C

```



```

C   UPDATE DIFFERENCES FOR NEXT STEP
C
C   DO 425 L = 1,NEQN
C       PHI(L,KP1) = YP(L) - PHI(L,1)
425   PHI(L,KP2) = PHI(L,KP1) - PHI(L,KP2)
C       DO 435 I = 1,K
C           DO 430 L = 1,NEQN
430       PHI(L,I) = PHI(L,I) + PHI(L,KP1)
435   CONTINUE
C
C   ESTIMATE ERROR AT ORDER K+1 UNLESS
C   IN FIRST PHASE WHEN ALWAYS RAISE ORDER,
C   ALWAYS RECIDED TO LOWER ORDER,
C   STEP SIZE NOT CONSTANT SO ESTIMATE UNRELIABLE
C
C       ERKP1 = 0.0
C       IF(KNEW .EQ. KM1 .OR. K.EQ. 12) PHASE1 = .FALSE.
C       IF(PHASE1) GO TO 450
C       IF(KNEW .EQ. KM1) GO TO 455
C       IF(KP1 .GT. NS) GO TO 460
C       DO 440 L = 1,NEQN
440       ERKP1 = ERKP1 = (PHI(L,KP2)/WT(L))**2
C       ERKP1 = ABSH*GSTR(KP1)*SQRT(ERKP1)
C
C   USING ESTIMATED ERROR AT ORDER K+1, DETERMINE
C   APPROPRIATE ORDER FOR NEXT STEP
C
C       IF(K .GT. 1) GO TO 445
C       IF(ERKP1 .GE. 0.5*ERK) GO TO 460
C       GO TO 450
445   IF(ERKM1 .LE. AMIN1(ERK,ERKP1)) GO TO 455
C       IF(ERKP1 .GE. ERK .OR. K .EQ. 12) GO TO 460
C
C   HERE ERKP1 .LT. ERK .LT. AMAX1(ERKM1,ERKM2) ELSE ORDER
C   WOULD HAVE BEEN LOWERED IN BLOCK 2.  THUS ORDER IS TO
C   BE RAISED
C
C   RAISE ORDER
C
450   K = KP1
C       ERK = ERKP1
C       GO TO 460
C
C   LOWER ORDER
455   K = KM1
C       ERK = ERKM1
C
C   WITH NEW ORDER DETERMINE APPROPRIATE STEP SIZE FOR
C   NEXT STEP
C

```

```

460  HNEW = H + H
      IF(PHASE1) GO TO 465
      IF(P5EPS .GE. ERK*TWO(K+1)) GO TO 465
      HNEW = H
      IF(P5EPS .GE. ERK) GO TO 465
      TEMP2 = K+1
      R = (P5EPS/ERK)**(1.0/TEMP2)
      HNEW = ABSH*AMAX1(0.5,AMAX1(0.9,R))
      HNEW = SIGN(AMAX1(HNEW,FOURU*ABS(X)),H)
465.  H = HNEW
      RETURN
      ***      END BLOCK 4      ***
      END
C*****
C  SUBROUTINE INTRP
C*****
      SUBROUTINE INTRP(X,Y,XOUT,YOUT,YPOUT,NEQN,KOLD,PHI,PSI)
      DIMENSION Y(NEQN),YOUT(NEQN),YPOUT(NEQN),PHI(NEQN,16),
        PSI(12)
      DIMENSION G(13),W(13),RHO(13)
      DATA G(1)/1.0/,RHO(1)/1.0/
      HI = XOUT - X
      KI = KOLD + 1
      KIPL = KI + 1

C
C  INITIALIZE W(*) FOR COMPUTING G(*)
C
      DO 5 I = 1,KI
        TEMPl = I
5      W(I) = 1.0/TEMPl
      TERM = 0.0

C
C  COMPUTE G(*)
C
      DO 15 J = 2,KI
        JM1 = J - 1
        PSIJM1 = PSI(JM1)
        GAMMA = (HI + TERM)/PSIJM1
        ETA = HI/PSIJM1
        LIMIT1 = KIPL - J
        DO 10 I = 1,LIMIT1
          W(I) = GAMMA*W(I) - ETA*W(I+1)
10      G(J) = W(1)
        RHO(J) = GAMMA*RHO(JM1)
15      TERM = PSIJM1

C
C  INTERPOLATE
C

```

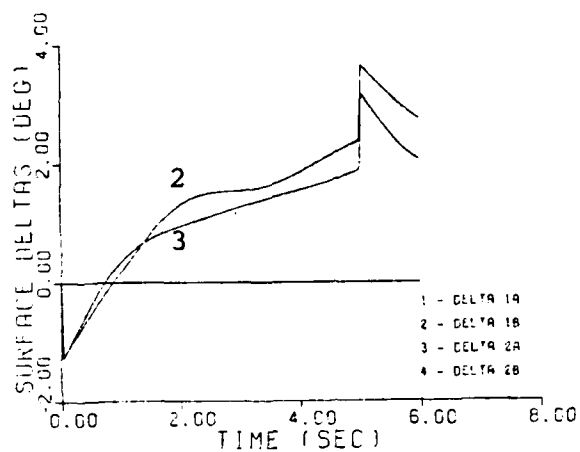


Fig. G-2(e). Control Surface Deflections for Pitch Rate Command, Flight Condition 3--CSC Mode 5

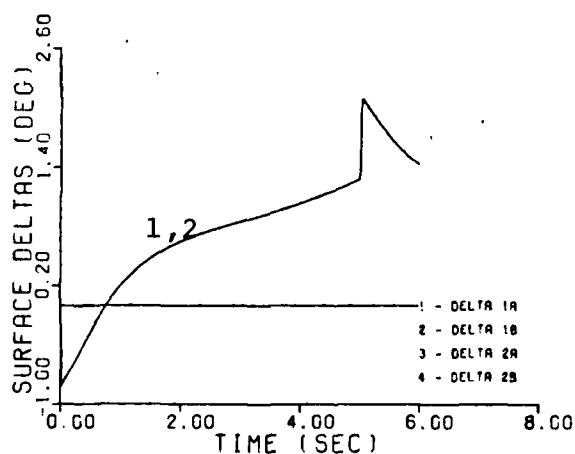


Fig. G-2(f). Control Surface Deflections for Pitch Rate Command, Flight Condition 3--CSC Mode 6

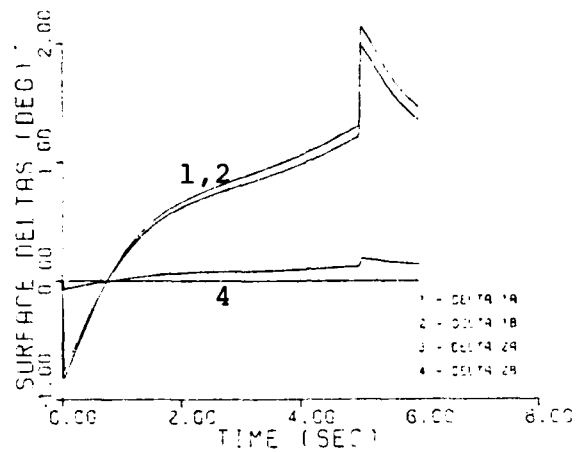


Fig. G-2(c). Control Surface Deflections for Pitch Rate Command, Flight Condition 3--CSC Mode 3

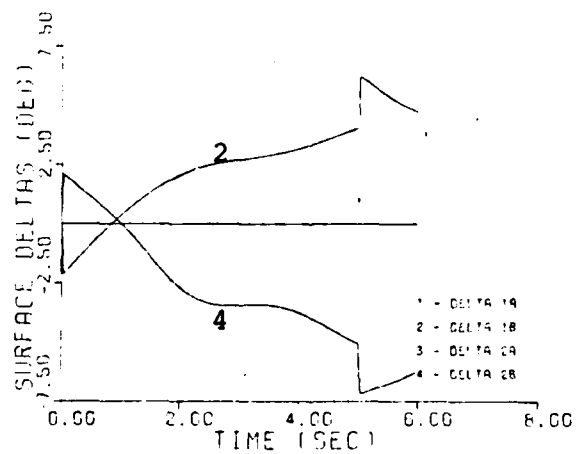


Fig. G-2(d). Control Surface Deflections for Pitch Rate Command, Flight Condition 3--CSC Mode 4

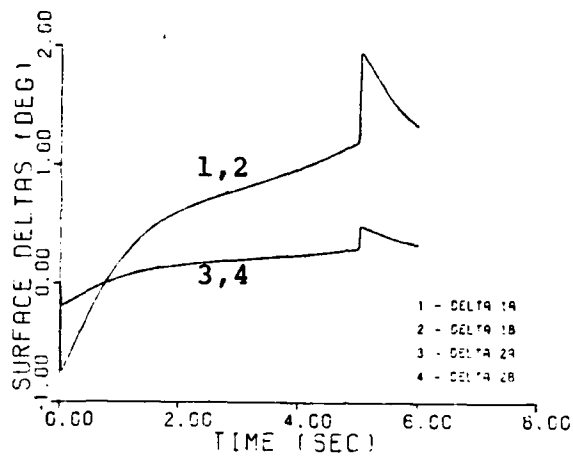


Fig. G-2(a). Control Surface Deflections for Pitch Rate Command, Flight Condition 3--CSC Mode 1

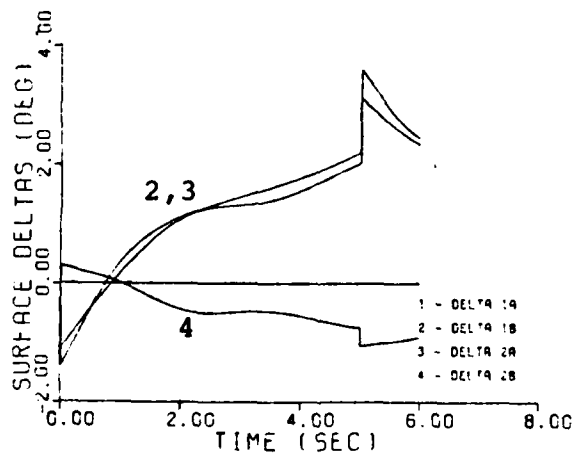


Fig. G-2(b). Control Surface Deflections for Pitch Rate Command, Flight Condition 3--CSC Mode 2

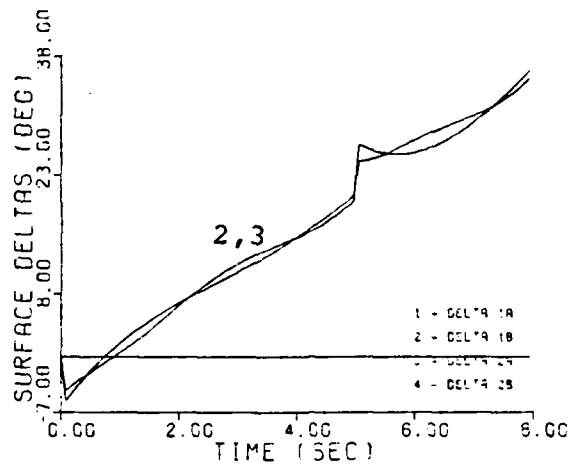


Fig. G-1(e). Control Surface Deflections for Pitch Rate Command, Flight Condition 2--CSC Mode 5

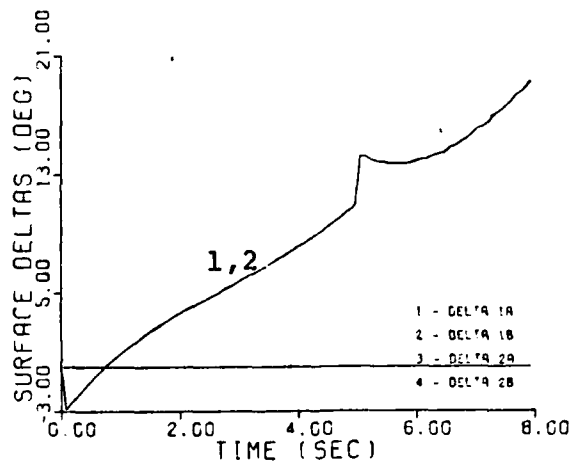


Fig. G-1(f). Control Surface Deflections for Pitch Rate Command, Flight Condition 2--CSC Mode 6

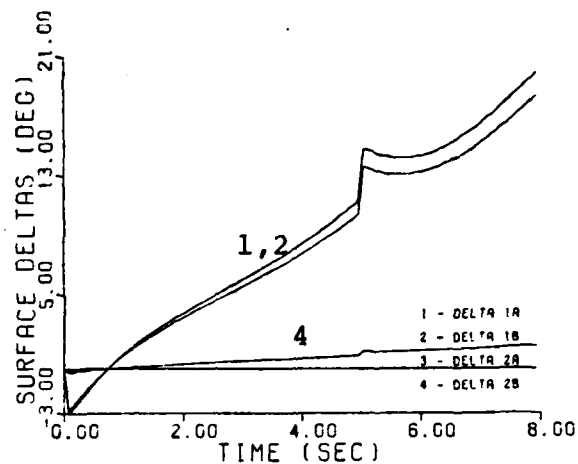


Fig. G-1(c). Control Surface Deflections for Pitch Rate Command, Flight Condition 2--CSC Mode 3

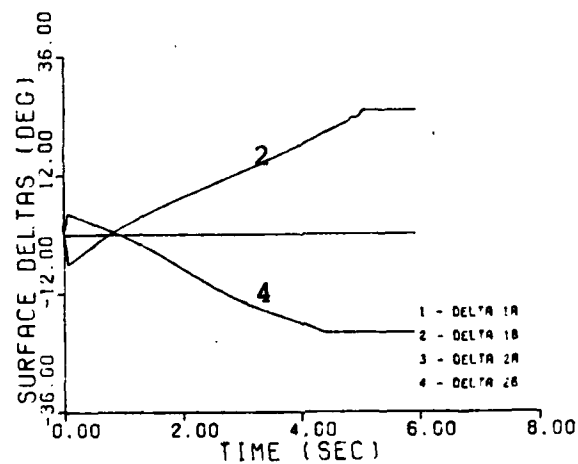


Fig. G-1(d). Control Surface Deflections for Pitch Rate Command, Flight Condition 2--CSC Mode 4

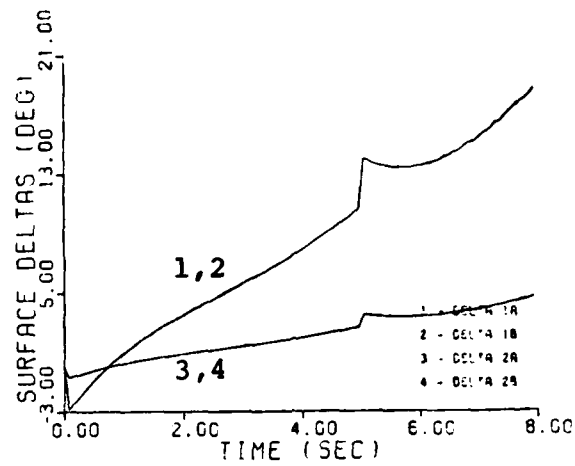


Fig. G-1(a). Control Surface Deflections for Pitch Rate Command, Flight Condition 2--CSC Mode 1

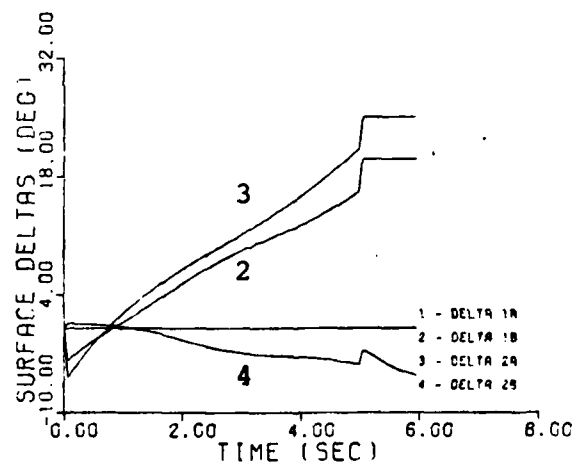


Fig. G-1(b). Control Surface Deflections for Pitch Rate Command, Flight Condition 2--CSC Mode 2



## Appendix G: Control Surface Deflections

In this appendix the control surface deflections due to  $q$  and  $p$  commands are illustrated for flight conditions two, three, and four.

C  
C  
C

P12

DATA VP12K/.2418,-.2118,-6.472,-5.85/  
DATA VP12Z1/0.,0.,0.,0./  
DATA VP12Z2/-.06537,-.006697,-.01254,-.03459/  
DATA VP12Z3/-.2589,-1.861,-1.646,-.6861/

C  
C

P22C

• DATA VP22K/-4.478,-17.446,-51.06,-14.168/  
DATA VP22Z1/0.,0.,0.,0./  
DATA VP22Z2/(-.205,.853),(-.219,1.607),(-.3541,2.927),  
+(-.4083,4.916)/  
DATA VP22Z3/(-.205 -.853),(-.219,-1.607),  
+(-.3541,-2.927),(-.4083,-4.916)/  
DATA VP22P1/-.6835,-.8265,-.02719,-.03448/  
DATA VP22P2/-.1041,-.07795,-2.697,-2.171/  
DATA VP22P3/(-.2741,1.909),(-.211,1.953),(-.391,2.962),  
+(-.4996,3.129)/  
DATA VP22P4/(-.2741,-1.909),(-.211,-1.953),  
+(-.391,-2.962),(-.4996,-3.129)/

C  
C  
C

P21

DATA VP21K/-4.284,-13.584,-51.72,-46.6/  
DATA VP21Z1/0.,0.,0.,0./  
DATA VP21Z2/(-.3017,1.562),(-.2442,2.101),  
+(-.3749,3.578),(-.3774,3.848)/  
DATA VP21Z3/(-.3017,-1.562),(-.2442,-2.101),  
+(-.3749,-3.578),(-.3774,-3.848)/

C  
C  
C

U12 = .25  
U21 = .25  
R1 = 10.  
R2 = 50.  
RETURN  
END

Zdata

## SUBROUTINE ZDATA

```

C
C-----  DIMENSION  -----
C      DIMENSION VP11K(4),VP11Z1(4),VP11Z2(4),VP11Z3(4)
C
C      DIMENSION VP12K(4),VP12Z1(4),VP12Z2(4),VP12Z3(4)
C
C      COMPLEX VP11P1(4),VP11P2(4),VP11P3(4),VP11P4(4)
C
C      DIMENSION VP21K(4),VP21Z1(4)
C      COMPLEX VP21Z2(4),VP21Z3(4)
C
C      DIMENSION VP22K(4),VP22Z1(4)
C      COMPLEX VP22Z2(4),VP22Z3(4)
C
C      DIMENSION VP22P1(4),VP22P2(4)
C      COMPLEX VP22P3(4),VP22P4(4)

```

```

C
C----- COMMON -----
COMMON /R/ R1,R2
COMMON /U/ U12,U21
C
COMMON /VP11Z/VP11K,VP11Z1,VP11Z2,VP11Z3
COMMON /VP11P/VP11P1,VP11P2,VP11P3,VP11P4
C
COMMON /VP12Z/VP12K,VP12Z1,VP12Z2,VP12Z3
C
COMMON /VP21Z/VP21K,VP21Z1,VP21Z2,VP21Z3
C
COMMON /VP22Z/VP22K,VP22Z1,VP22Z2,VP22Z3
COMMON /VP22P/VP22P1,VP22P2,VP22P3,VP22P4

```

## PLANT DATA

```
C      P11
      DATA VP11K/ -2.236, -5.862, -24.06, -32.9/
      DATA VP11Z1/ 0., 0., 0., 0./
      DATA VP11Z2/ -.01822, -.01004, -.01263, -.02996/
      DATA VP11Z3/ -.4568, -.5502, -1.51, -1.097/
      DATA VP11P1/ (-1.3, .0.), (-2.028, 0.), (-3.223, 0.),
+      (-.01516, .02343)/
      DATA VP11P2/ (.3633, 0.), (1.167, 0.), (.9645, 0.),
+      (-.01516, -.02343)/
      DATA VP11P3/ (-.07683, .2065), (-.006472, .07803),
+      (-.007553, .05384), (-.8012, 6.592)/
      DATA VP11P4/ (-.07683, -.2065), (-.006472, -.07803),
+      (-.007553, -.05384), (-.8012, -6.592)/
```

C

D1A = G1 + G12  
D1B = G1 - G12  
D2A = G21 + G2  
D2B = G21 - G2

C

IF (IS1A .EQ. 0) D1A = 0.  
IF (IS1B .EQ. 0) D1B = 0.  
IF (IS2A .EQ. 0) D2A = 0.  
IF (IS2B .EQ. 0) D2B = 0.

C

C----- SATURATION -----

C

IF (D1A .GT. 25.) D1A = 25.  
IF (D1A .LT. -25.) D1A = -.25.  
IF (D1B .GT. 25.) D1B = 25.  
IF (D1B .LT. -25.) D1B = -25.

C

IF (D2A .GT. 20.) D2A = 20.  
IF (D2A .LT. -20.) D2A = -20.  
IF (D2B .GT. 20.) D2B = 20.  
IF (D2B .LT. -20) D2B = -20.

C

RETURN  
END

```

C----- F(S) -----
C
  YP(17) = R1 - F1P*Y(17)
  F1 = F1K*Y(17)
C
  YP(18) = R2 - F2P*Y(18)
  F2 = F2K*Y(18)
C
C
C----- G1(S) -----
C
  E1 = F1 - C1
C
  YP(19) = E1
  G1A = Y(19)
C
  YP(20) = G1A - G1P1*Y(20)
  1 = YP(20) + G1Z1*Y(20)
C
  YP(21) = G1B - G1P2*Y(21)
  G1C = YP(21) + G1Z2*Y(21)
  YP(22) = Y(23)
  YP(23) = G1C - G1P3*Y(23) - G1P4*Y(22)
  G1D = YP(23) + G1Z3*Y(23) + G1Z4*Y(22)
  G1 = G1K*G1D
C
C
C----- G2(S) -----
C
  E2 = F2 - C2
C
  YP(24) = E2
  G2A = Y(24)
C
  YP(25) = G2A - G2P1*Y(25)
  G2B = YP(25) + G2Z1*Y(25)
C
  YP(26) = G2B - G2P2*Y(26)
  G2C = YP(26) + G2Z2*Y(26)
C
  YP(27) = Y(28)
  YP(28) = G2C - G2P3*Y(28) - G2P4*Y(27)
C
  G2D = YP(28) + G2Z3*Y(28) + G2Z4*Y(27)
C
  G2 = G2K*G2D
C
C
C----- DELTA -----
C
  G12 = U12*G2
  G21 = U21*G1

```

```

C
    C11 = P11K*D11B
C
C----- P12(S) -----
C
    D12 = D2A + D2B
C
    YP(5) = Y(6)
    YP(6) = D12 - P11P1*Y(6) - P11P2*Y(5)
    D12A = Y(6) + P12Z1*Y(5)
C
    YP(7) = Y(8)
    YP(8) = D12A - P11P3*Y(8) - P11P4*Y(7)
    D12B = YP(8) + P12Z2*Y(8) + P12Z3*Y(7)
C
    C12 = P12K*D12B
C
C----- P21(S) -----
C
    D21 = D1A - D1B
    YP(9) = Y(10)
    YP(10) = D21 - P22P1*Y(10) - P22P2*Y(9)
    D21A = Y(10) + P21Z1*Y(9)
C
    YP(11) = Y(12)
    YP(12) = D21A - P22P3*Y(12) - P22P4*Y(11)
    D21B = YP(12) + P21Z2*Y(12) + P21Z3*Y(11)
C
    C21 = P21K*D21B
C
C----- P22(S) -----
C
    D22 = D2A - D2B
C
    YP(13) = Y(14)
    YP(14) = D22 - P22P1*Y(14) - P22P2*Y(13)
    D22A = Y(14) + P22Z1*Y(13)
C
    YP(15) = Y(16)
    YP(16) = D22A - P22P3*Y(16) - P22P4*Y(15)
    D22B = YP(16) + P22Z2*Y(16) + P22Z3*Y(15)
C
    C22 = P22K*D22B
C
C----- C -----
    C1 = C11 + C12
    C2 = C21 + C22
C
C

```

Derv

```

      SUBROUTINE DERV(T,Y,YP)
C
C   RECONFIGURATION SYSTEM
C
      DIMENSION Y(28),YP(28)
C***** COMMON *****
C
      COMMON /P11Z/ P11K,P11Z1,P11Z2,P11Z3
      COMMON /P11P/ P11P1,P11P2,P11P3,P11P4
C
      COMMON /P12Z/ P12K,P12Z1,P12Z2,P12Z3
C
      COMMON /P21Z/ P21K,P21Z1,P21Z2,P21Z3
      COMMON /P21P/ P21P1,P21P2,P21P3,P21P4
C
      COMMON /P22Z/ P22K,P22Z1,P22Z2,P22Z3
      COMMON /P22P/ P22P1,P22P2,P22P3,P22P4
C
      COMMON /F/ F1K,F1P,F2K,F2P
C
      COMMON /G1Z/G1K,G1Z1,G1Z2,G1Z3,G1Z4
      COMMON /G1P/G1P1,G1P2,G1P3,G1P4
C
      COMMON /G2Z/G2K,G2Z1,G2Z2,G2Z3,G2Z4
      COMMON /G2P/G2P1,G2P2,G2P3,G2P4
C
      COMMON /R/ R1,R2
      COMMON /U/ U1,U2
      COMMON /C/ C1,C2
      COMMON /D/ D1A,D1B,D2A,D2B
      COMMON /IS/ IS1A,IS1B,IS2A,IS2B
      COMMON /M/ F1,F2,G1,G2,E1,E2,D11,D12,D21,D22,
      C11,C12,C21,C22
C
C----- P11(S) -----
C
      D11 = D1A + D1B
C
      YP(1) = Y(2)
      YP(2) = D11 - P11P1*Y(2) - P11P2*Y(1)
      D11A = Y(2) + P11Z1*Y(1)
C
      YP(3) = Y(4)
      YP(4) = D11A - P11P3*Y(4) - P11P4*Y(3)
      D11B = YP(4) + P11Z2*Y(4) + P11Z3*Y(3)
```

```

      DO 20 L = 1,NEQN
        YPOUT(L) = 0.0
20     YOUT(L) = 0.0
      DO 30 J = 1,KI
        I = KIP1 - J
        TEMP2
= G(I)      TEMP3 = RHO(I)
      DO 25 L = 1,NEQN
        YOUT(L) = YOUT(L) + TEMP2*PHI(L,I)
        YPOUT(L) = YPOUT(L) + TEMP3*PHI(L,I)
30     CONTINUE
      DO 35 L = 1,NEQN
35     YOUT(L) = Y(L) + HI*YOUT(L)
      RETURN
C
      END

```



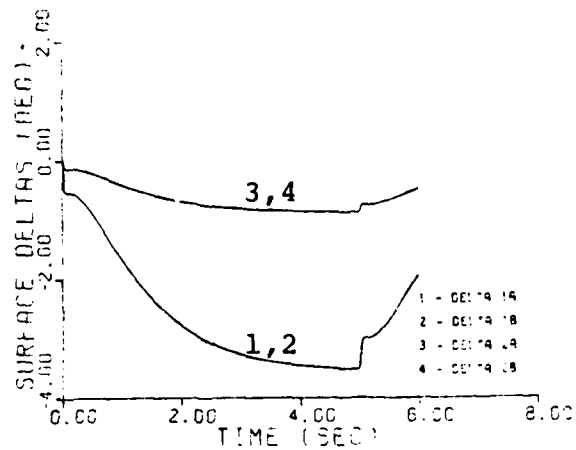


Fig. G-3(a). Control Surface Deflections for Pitch Rate Command, Flight Condition 4--CSC Mode 1

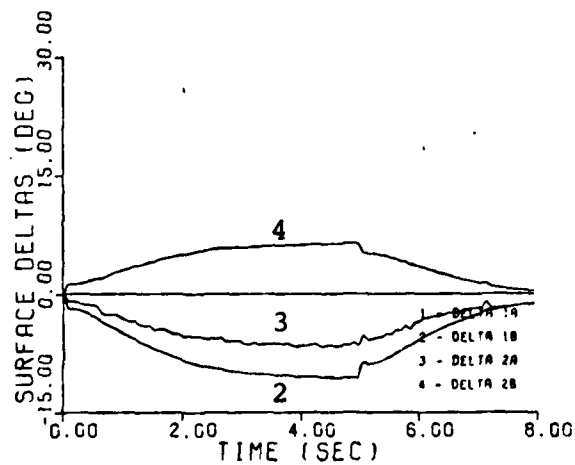


Fig. G-3(b). Control Surface Deflections for Pitch Rate Command, Flight Condition 4--CSC Mode 2

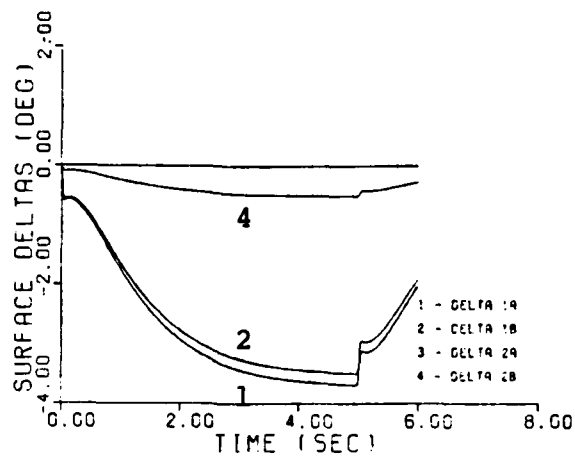


Fig. G-3(c). Control Surface Deflections for Pitch Rate Command, Flight Condition 4--CSC Mode 3

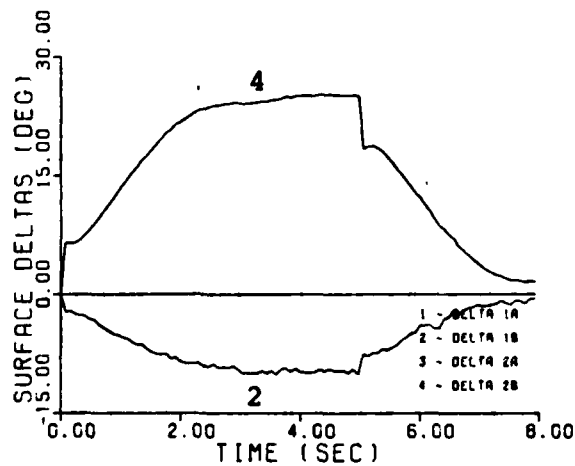


Fig. G-3(d). Control Surface Deflections for Pitch Rate Command, Flight Condition 4--CSC Mode 4

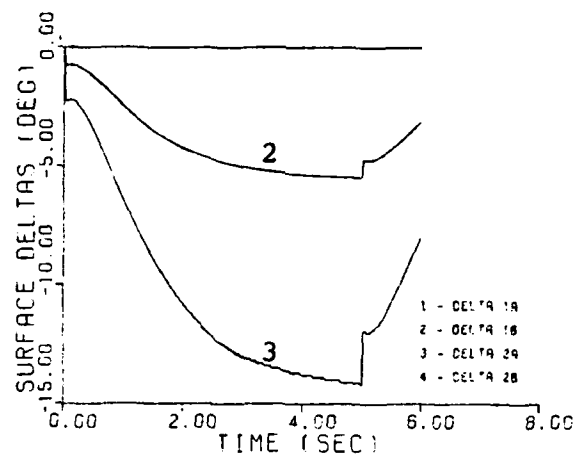


Fig. G-3(e). Control Surface Deflections for Pitch Rate Command, Flight Condition 4--CSC Mode 5

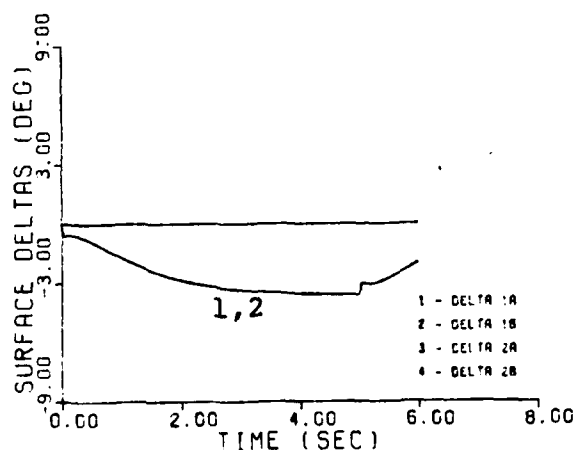


Fig. G-3(f). Control Surface Deflections for Pitch Rate Command, Flight Condition 4--CSC Mode 6

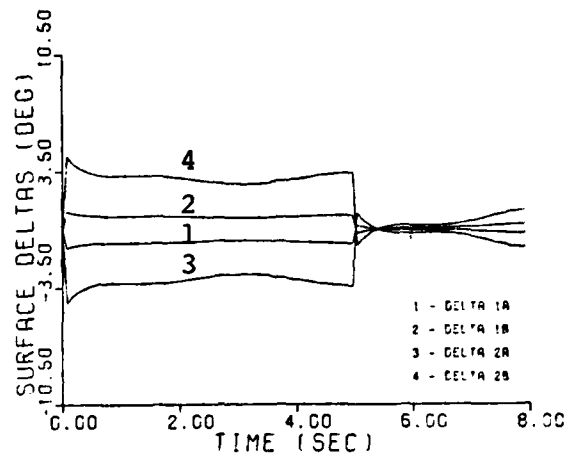


Fig. G-4(a). Control Surface Deflections for Roll Rate Command, Flight Condition 2--CSC Mode 1

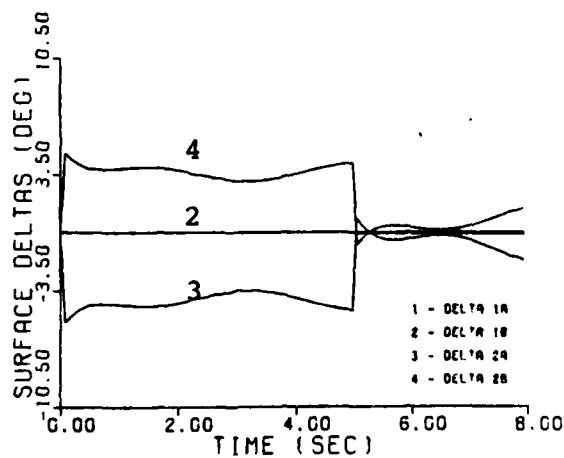


Fig. G-4(b). Control Surface Deflections for Roll Rate Command, Flight Condition 2--CSC Mode 2

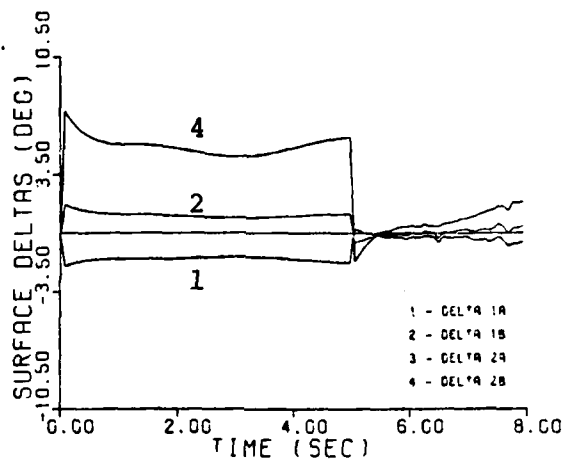


Fig. G-4(c). Control Surface Deflections for Roll Rate Command, Flight Condition 2--CSC Mode 3

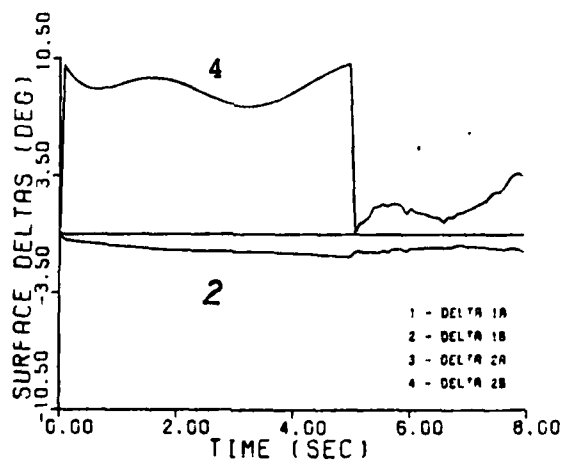


Fig. G-4(d). Control Surface Deflections for Roll Rate Command, Flight Condition 2--CSC Mode 4

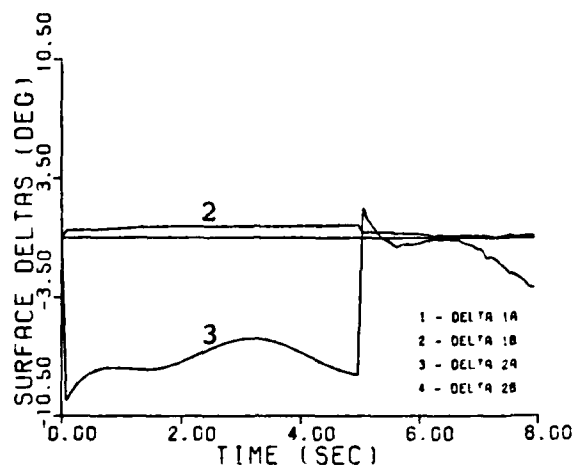


Fig. G-4(e). Control Surface Deflections for Roll Rate Command, Flight Condition 2--CSC Mode 5

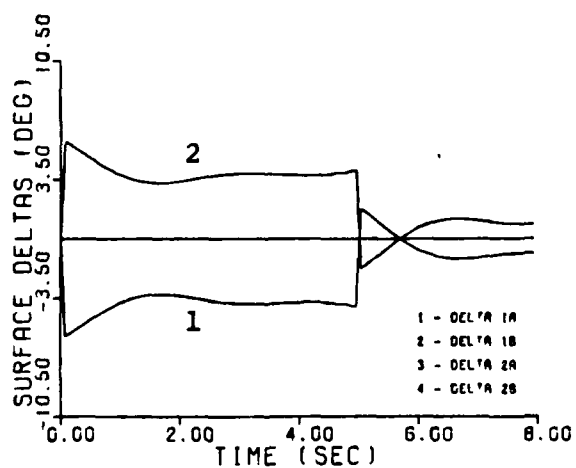


Fig. G-4(f). Control Surface Deflections for Roll Rate Command, Flight Condition 2--CSC Mode 6

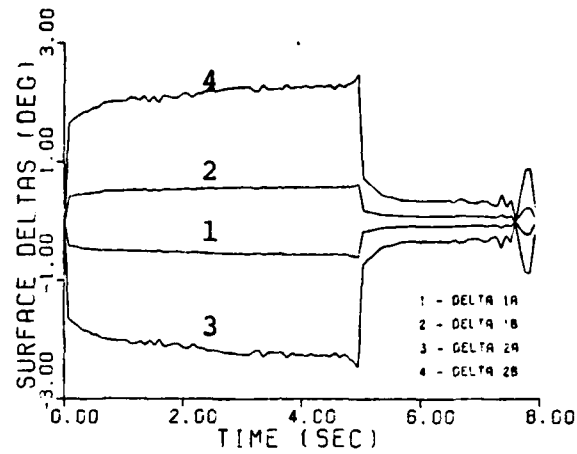


Fig. G-5(a). Control Surface Deflections for Roll Rate Command, Flight Condition 3--CSC Mode 1

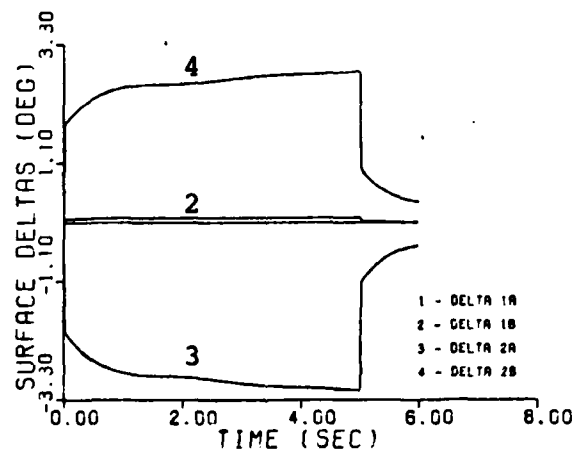


Fig. G-5(b). Control Surface Deflections for Roll Rate Command, Flight Condition 3--CSC Mode 2

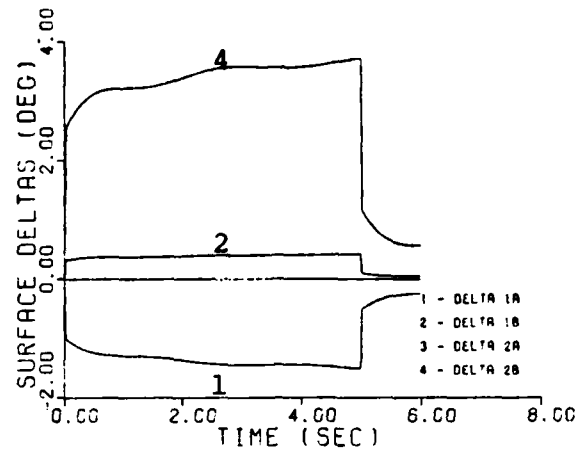


Fig. G-5(c). Control Surface Deflections for Roll Rate Command, Flight Condition 3--CSC Mode 3

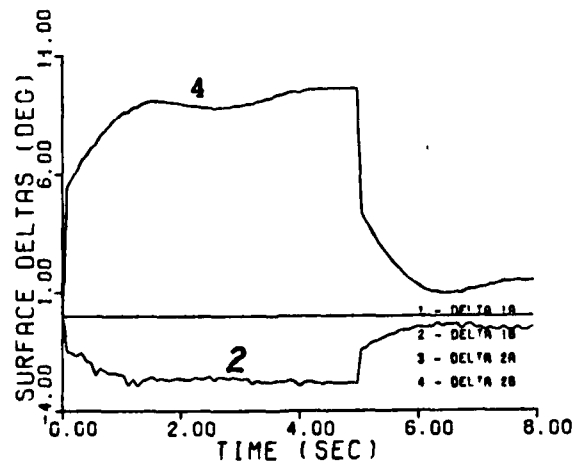


Fig. G-5(d). Control Surface Deflections for Roll Rate Command, Flight Condition 3--CSC Mode 4



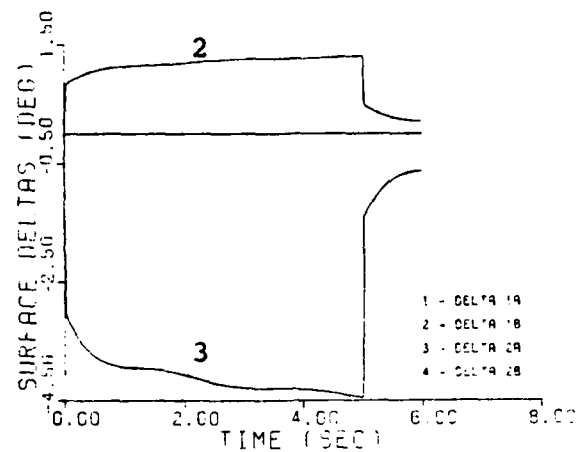


Fig. G-5(e). Control Surface Deflections for Roll Rate Command, Flight Condition 3--CSC Mode 5

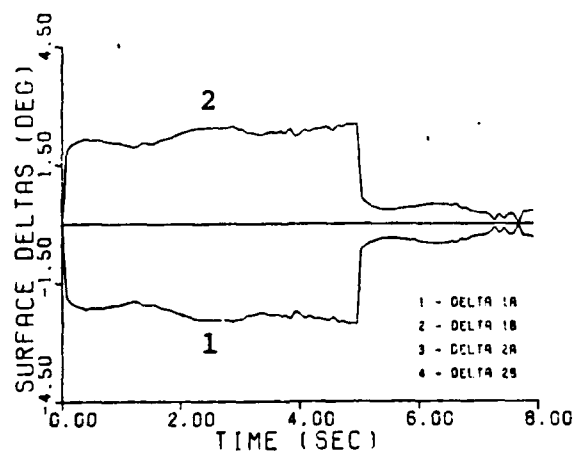


Fig. G-5(f). Control Surface Deflections for Roll Rate Command, Flight Condition 3--CSC Mode 6

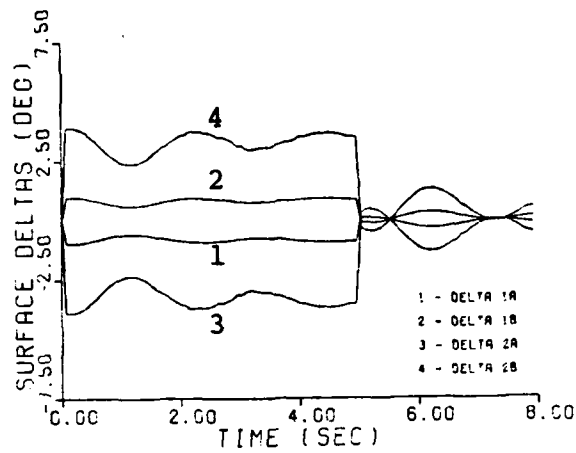


Fig. G-6(a). Control Surface Deflections for Roll Rate Command, Flight Condition 4--CSC Mode 1

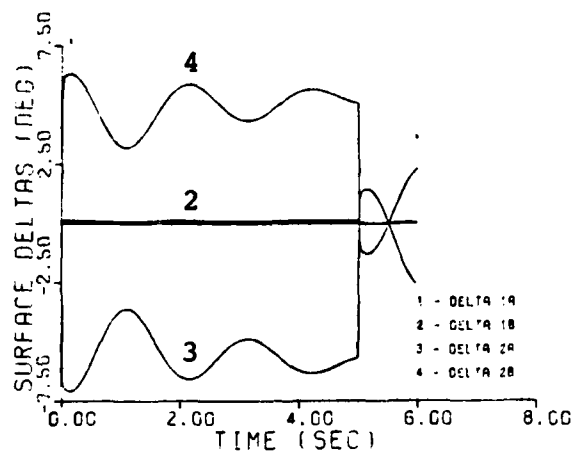


Fig. G-6(b). Control Surface Deflections for Roll Rate Command, Flight Condition 4--CSC Mode 2

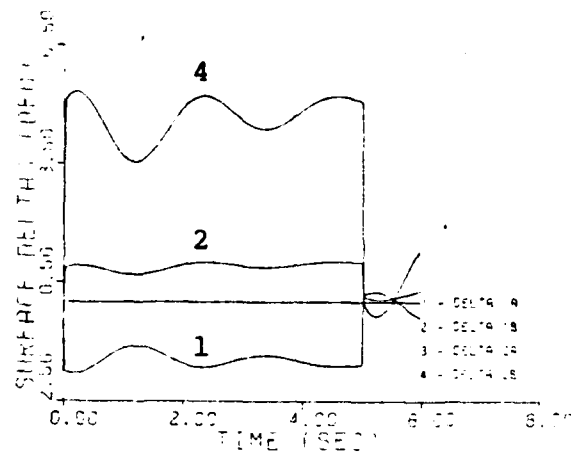


Fig. G-6(c). Control Surface Deflections for Roll Rate Command, Flight Condition 4--CSC Mode 3

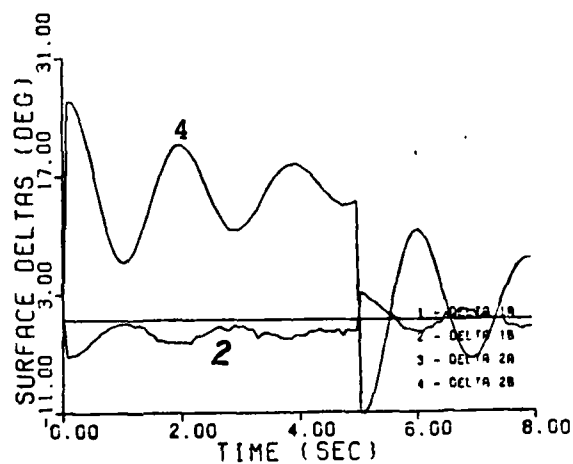


Fig. G-6(d). Control Surface Deflections for Roll Rate Command, Flight Condition 4--CSC Mode 4

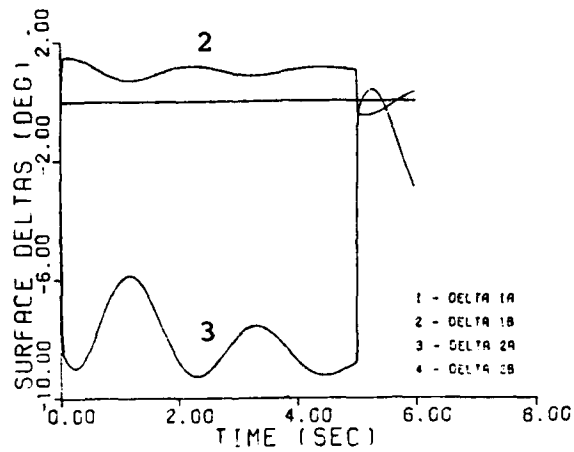


Fig. G-6(e). Control Surface Deflections for Roll Rate Command, Flight Condition 4--CSC Mode 5

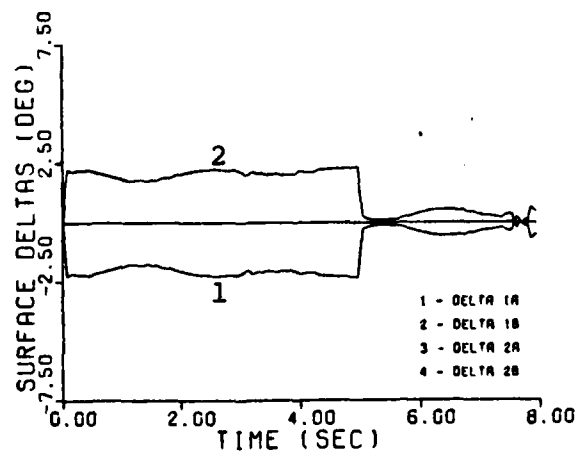


Fig. G-6(f). Control Surface Deflections for Roll Rate Command, Flight Condition 4--CSC Mode 6

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### Vita

Major Phillip B. Arnold attended the University of Oklahoma from September 1965 until May 1970. He graduated with the degree of Bachelor of Science in Electrical Engineering, and received a reserve commission through the Air Force ROTC program in May 1970. In October 1971 he was awarded his pilot's wings, and the next six years were spent as an interceptor pilot in the Air Defense Command flying the F-106. From May 1977 until June 1980 he served as an exchange officer with the Canadian Armed Forces in British Columbia, Canada flying the CF-101. In June 1981 he graduated from the USAF Test Pilot School at Edwards AFB, California. The two years prior to coming to the Air Force Institute of Technology (AFIT) were spent in the Tactical Air Command at Tyndall AFB, Florida as a test pilot in an operational test and evaluation squadron.

Major Arnold has a senior pilot rating and more than 3000 hours of flying time. His assignment following graduation from AFIT is to the USAF Test Pilot School where he will be on the staff.

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Quantitative theory developed by Dr. Isaac Horowitz of the Weizmann Institute of Science, Israel is used to develop control laws for the AFTI/F-16 with a reconfigurable flight control system. Compensators are synthesized to control pitch rate and roll rate through individually controlled elevators and flaperons. Robust control of these variables is required over a large portion of the flight envelope despite flight control surface failures.

Linearized aerodynamic data are used to develop the aircraft model in state-variable format. The longitudinal and lateral-directional equations are coupled in the control matrix. Individual control of the elevators and flaperons is obtained by dividing the dimensionalized control derivatives for a control surface pair in half and assigning each surface of the pair one-half of the total derivative value. The system with individually controlled surfaces represents a four input-two output system which is transformed into an equivalent two input-two output system for each control surface configuration and flight condition. Quantitative feedback theory is then applied to the equivalent systems. *Original - supplied keywords included. →/cat p.*

Fixed-compensators and prefilters are synthesized for the pitch rate and roll rate channels, and a digital simulation is conducted at four points in the flight envelope with up to two simultaneous surface failures, as well as, for the unimpaired aircraft. Results of the simulation show that the compensators provide robust control for each control surface configuration at each flight condition. Compensator designs are also synthesized to provide only stability when three of the four surfaces are failed.

The theory developed by Dr. Horowitz is found to be effective in designing for robustness despite large uncertainty arising from control surface failures, and plant parameter variation. Flight control research using quantitative feedback theory should be continued. The fixed-compensator design approach has potential to significantly reduce the complexity of a reconfigurable flight control system. A computer-aided design package for quantitative feedback theory should be developed to facilitate design work.



**END**

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